

A Brazilian Portuguese work on MetaPost, and how mathematics is embedded in it

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Abstract

This article presents a summary of my final work [1] for the master's degree course in Mathematics, presented on November 1, 2018. This work shows an introduction to the basic commands of the MetaPost language. For this, some mathematical concepts are explained, as they are part of its programming. It is shown how these concepts can be used to create high-quality figures used in areas of mathematics such as geometry and the study of graphs of real functions.

Introduction

Written communication is the basis for teaching and learning in various areas of knowledge. Writing a legible and elegant mathematical work was not so simple in the old days without the computational tools available today. The ability to expose knowledge in accordance with technical requirements, or even personal norms, moved people to create typographic writing systems that included mathematical symbols and concepts. MetaPost is a very useful tool for this.

When I needed a research topic for my master's thesis, my advisor, Professor Elisabete Freitas, introduced me to MetaPost and said that it would be very nice to study some of the mathematics behind it. She said she uses MetaPost whenever she needs to create pictures for her work on T_EX and that it has powerful tools that can contribute to the work of Mathematics teachers. So, I met this language and started studying its pedagogical use.

What did I study?

Chapter 1 of the dissertation presents some commands for creating basic figures for an interested reader who has never heard of MetaPost. To identify the positions in a drawing, MetaPost uses the one-to-one correspondence between the points of a plane and the set of ordered pairs of real numbers. Mathematics!

In this initial chapter, the reader learns about the commands `draw`, `drawdot`, `cycle`, paths with `--` and `...`, style commands that change the width, size, or shape of a solid or dashed line, `label`, `dotlabel`, `drawarrow`, `drawblarrow`, some data types, commands for colors, filling and mathematical operators. It also contains the conversion table of the units of measurement used in MetaPost, as shown in Table

1 and a short tutorial on how to insert a MetaPost figure into a T_EX document.

Unit of measurement	Conversion
pt	0,035145 cm
bp	0,035278 cm
in	2,54 cm
pc	0,423333 cm
mm	0,1 cm
cc	0,451167 cm
dd	0,0376 cm

Table 1: Conversion table of the units of measurement used in MetaPost

Some mathematical concepts of vectors and geometry can be used to make drawings in MetaPost. MetaPost has vector operators and is able to understand the data of the type pair as vectors. Chapter 2 addresses the mathematical basis for this, as well as some applications of these operations in geometry.

This chapter defines vectors and shows some vector properties. It presents the concept of colinearity and dot product (inner product) of two vectors. An application of these concepts is used to show how to plot the altitude of a triangle with the `dotprod` operator, as seen in fig. 1.

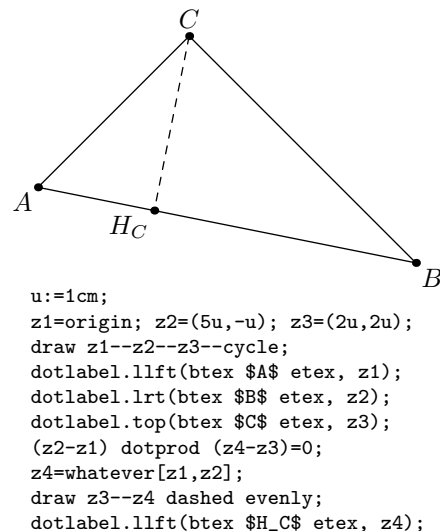


Figure 1: Triangle ABC with relative altitude drawn on side AB

The second chapter also shows the mathematics behind the drawing of a regular polygon and the four triangle centers: centroid, circumcenter, incenter and orthocenter. In addition, it shows how to draw them with MetaPost (figs. 2-6).

Chapter 3 discusses transformations in the plane, as well as features that allow you to use them in MetaPost to draw various graphics. Some applications

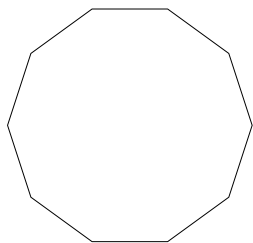


Figure 2: Decagon of side 1 cm.

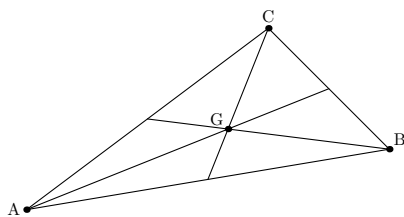


Figure 3: Medians and the centroid G of ABC.

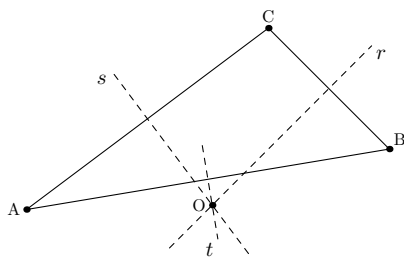


Figure 4: Perpendicular bisectors and the circumcenter O of ABC.

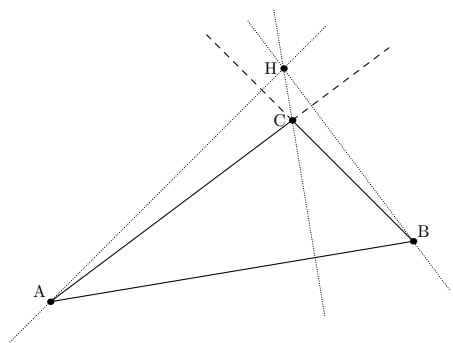


Figure 5: Altitudes and the orthocenter H of ABC.

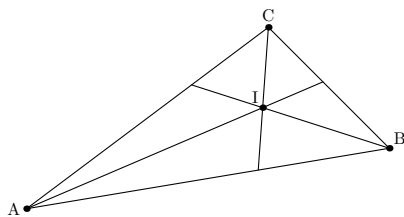


Figure 6: Angle bisectors and the incenter I of ABC.

of these transformations are also presented for the sketch of circles and ellipses.

A transformation in the plane \mathbb{R}^2 is a function $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, i.e, a correspondence that associates with each point P of the plane another point $P_1 = T(P)$ of the plane, called its image by T . In MetaPost, transformations are performed using the `transform` type. With them, along with the `fullcircle` command, you can draw circles and ellipses, as shown in figures 7 and 8.

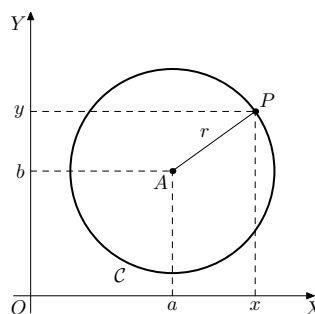


Figure 7: Circle C of center A and radius r

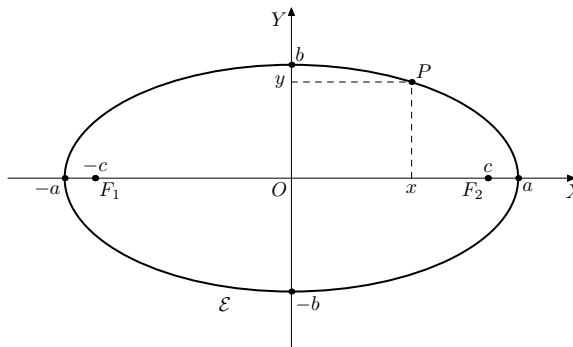
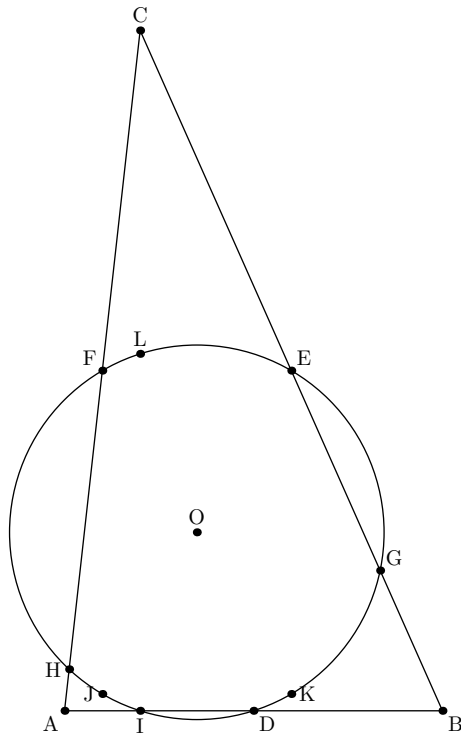


Figure 8: Ellipse E of center O, focuses F_1 and F_2 and major axis $2a$

Although it may seem obvious that using the `scaled`, `xscaled`, `yscaled`, and `shifted` transforms generates the circles and ellipses from the initial circle `fullcircle`, the mathematics behind this concept is accurate and, if I may say, particularly beautiful. These results are exemplified with two interesting problems of geometry: the nine-point circle and the problem of tracing the ellipse tangent to the triangle with one of its focuses an inner point of the triangle, as shown in figures 9 and 10.



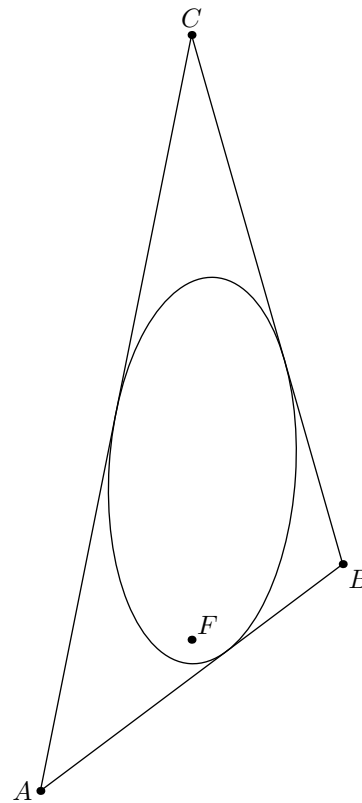
```

numeric u; u=1cm;
pair A,B,C,D,E,F,G,H,I,J,K,L,O,X;
A=(0,u);
B=(5u,u);
C=(u,10u);
D=.5[A,B];
E=.5[B,C];
F=.5[A,C];
G=whatever[C,B];
H=whatever[A,C];
I=whatever[B,A];
(A-G) dotprod (B-C) =
(B-H) dotprod (C-A) =
(C-I) dotprod (A-B) = 0;
X = whatever[A,G] = whatever[B,H];
J=.5[A,X]; K=.5[B,X]; L=.5[C,X];
(0-0.5(D+I)) dotprod (D-I) =
(0-0.5(G+K)) dotprod (G-K) = 0;
draw A--B--C--cycle;
draw fullcircle
scaled (2*abs(O-D)) shifted O;

```

Figure 9: Nine-point circle

Chapter 4 deals with the elaboration of graphs of continuous functions defined in intervals of the line. Initially a summary is made with some propositions about continuous functions. Next, some MetaPost techniques are outlined to sketch function graphs through macros (fig. 11). The techniques in this chapter discuss how to use a loop to calculate coordinates of graph points, how to cut a piece of the graph you want to see, how to use or create macros for non-predefined math functions in MetaPost, and how to plot graphs with asymptotes.



```

numeric u,a,b,c,theta; transform T;
pair A,B,C,F[],G[],O,P[]; path E;
u=1cm; A=(-u,-3u); B=(3u,0); C=(u,7u);
draw A--B--C--cycle; F1=(u,-u); F3=whatever[A,C];
F4=whatever[A,B]; F5=whatever[C,B];
(F3-F1) dotprod (A-C) = (F4-F1) dotprod (A-B) =
(F5-F1) dotprod (C-B) = 0;
G1 = F1 rotatedaround (F3,180);
G2 = F1 rotatedaround (F4,180);
G3 = F1 rotatedaround (F5,180);
(F2-0.5(G2+G3)) dotprod (G2-G3) =
(F2-0.5(G1+G2)) dotprod (G1-G2) = 0;
P1=whatever[A,C]=whatever[F2,G1]; O=0.5[F1,F2];
2a=abs(F1-P1)+ abs(F2--P1); c=abs(F1--O); b=a++c;
theta=angle(F2-F1); T:= identity xscaled 2a
yscaled 2b rotated theta shifted O;
E:= fullcircle transformed T; draw E;

```

Figure 10: Ellipse inscribed in the triangle ABC with one focus at F

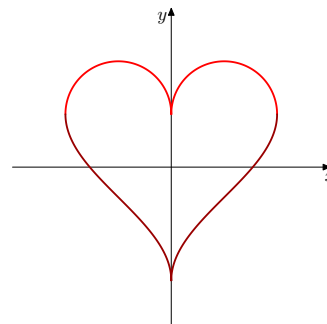
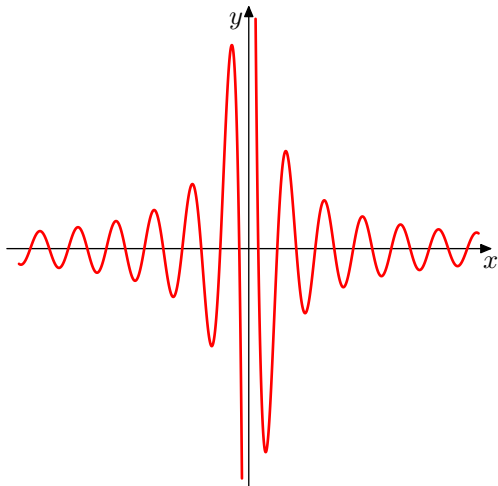


Figure 11: Functions $f(x) = 1 + \sqrt{1 - |x - 1|^2}$ and $g(x) = 1 + \arccos(1 - |x|) - \pi$ (grayscale for print)

For instance, the graph of figure 12 was sketched using some of these techniques. First, some macros are defined for the calculation of the cosine function. Then the limits of the axes of the Cartesian plane and the limits of the graph of the function are defined. Since the function is defined in two disjoint intervals, a loop is used to calculate 1000 points of each interval. Then another loop is used to define the two smooth curves through these points. The last step is to draw these two curves respecting the preset limits.



```
def pi = arclength fullcircle enddef;
numeric radian; radian := 180/pi;
vardef cos primary x = (cosd(x*radian)) enddef;
numeric u, minx, maxx, miny, maxy; path q;
u:=0.8cm; minx:=-4; maxx:=4; miny:=-4; maxy:=4;
drawarrow ((minx,0)--(maxx,0)) scaled u;
drawarrow ((0,miny)--(0,maxy)) scaled u;
label.bot (btex $x$ etex, (maxx*u,0));
label.llft (btex $y$ etex, (0,maxy*u));
q:=((minx,miny)--(maxx,miny)--(maxx,maxy)--
(minx,maxy)--cycle) scaled 0.95;
numeric h,n; path p[];
n:=1000; a:=0.1; b:=maxx; h:=(b-a)/n;
for i=0 upto n: x[i]:=a+i*h;
y[i]:=cos(10*x[i])/x[i]; endfor;
p[1]=z[0] for j=1 upto n: ..z[j] endfor;
a:=minx; b:=-0.1; h:=(b-a)/n;
for i=0 upto n: x[i]:=a+i*h;
y[i]:=cos(10*x[i])/x[i]; endfor;
p[2]=z[0] for j=1 upto n: ..z[j] endfor;
pickup pencircle scaled 1bp;
draw (p1 cutafter q cutbefore q)
scaled u withcolor red;
draw (p2 cutafter q cutbefore q)
scaled u withcolor red;
```

Figure 12: Function $f(x) = \frac{\cos 10x}{x}$
(grayscale for print)

Final considerations

MetaPost is a powerful tool for using mathematical knowledge in the preparation of figures. The images generated through this language have a satisfactory professional level for what is expected of an academic work. People who teach and research mathematics can improve the presentation of their work and guide their students in the application of acquired mathematical knowledge. Thus, I hope that this work may, in a way, have contributed to the development of mathematics teachers and, consequently, to the development of their students.

I am part of a research group of the Institute of Mathematics of the Federal University of Mato Grosso do Sul, called “Fundamentals of Mathematics for High School and the use of the language MetaPost”. This project proposes to provide material for the high school curriculum using \TeX and MetaPost. We also intend to disseminate this language in the academic environment of Brazil, since there is very little work on MetaPost in Portuguese. If you would like to collaborate, please contact us via my email address.

References

- [1] CANDIA, Estevão V. *Matemática e o MetaPost*. Master’s degree thesis. UFMS, 2018. Available at http://sca.proformat-sbm.org.br/sca_v2/get_tcc3.php?id=160350039.

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