A short history of the Lucida math fonts

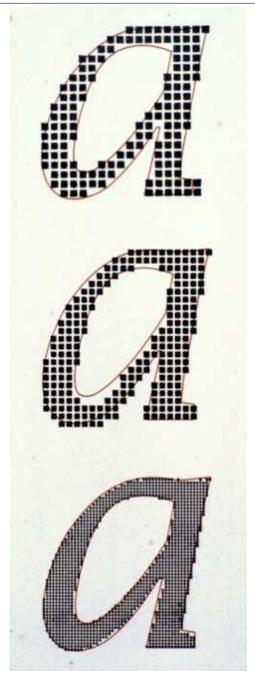
Charles Bigelow

This talk is about the development of Lucida math fonts from the beginnings of Lucida in the early 1980s to the most recent Lucida OpenType math fonts of 2011 and later. The images included here, and more, are available online at http://tug.org/tug2016/slides/bigelow-lucidamath.pdf.

1. The above shows a sampling of characters in Lucida Math fonts. There are letter-like symbols and also different shapes and sizes of arrows, operators, relations, and delimiters.

Typography involves several kinds of harmony between graphical elements. Weight, height, width, stroke thickness, proportions, shapes, orientation, position, spacing, and stylistic features such as serifs, are the main relationships that may have to be harmonized. In the typography of mathematics, many symbols are derived from letters but have become separate semantic elements, not units necessarily combined into words. Within the formal language of mathematics, symbols become ideograms—representing ideas — and logograms — spoken as a word or phrase. Naturally, the spoken version changes from language to language, while the meaning of the symbol is constant. Moreover, symbols for mathematics may need adjustment for optical scale because some may be used large and small. And, of course they must be legible.

2. What were the principles on which Lucida was based? Although we were designing a typeface family for emerging technologies—laser printing and screen display—we tended to look backwards to the history of letter forms, in traditional handwriting or calligraphy, which goes back more than two thousand years for Latin scripts and for more than five hundred years before that for Greek.



3. We wanted the forms of the Lucida letters to be related to traditional scribal handwriting, but not necessarily "calligraphy" in its sense of "beautiful

writing". Instead we aimed for simplicity because our goal was easy reading under difficult imaging conditions.

Our goal was to adapt traditional forms for simpler rendering at a range of resolutions. We developed the letters in outline formats, principally using Peter Karow's Ikarus system, but the outlines were then converted to bitmaps, either on-the-fly in printers or pre-rasterized for screens, in the technology of the early 1980s. Laser printers of the late 1970s and early 1980s had addressable resolutions around 240 to 300 dots per inch, and bitmap display screen resolutions were around 70 to 75 pixels per inch on screen.

We made many experiments constructing letters from bitmaps, and then used a few digital systems of the early 1980s to study the different pixel patterns when letters were rasterized at different resolutions. At high resolutions, the letters looked like analog typefaces, but at low resolutions, the letters became minimalist aggregations of dots, and many distinguishing features that separated one type design from another were obscured by the rough, rasterized stair-cased shapes and by data loss.

ABCDEFGHIJKLMNOPQRSTUVWXYZ abcdefghijklmnopqrstuvwxyz 1234567890

ABCDEFGHIJKLMNOPQRSTUVWXYZ abcdefghijklmnopqrstuvwxyz 1234567890

ABCDEFGHIJKLMNOPQRSTUVWXYZ abcdefghijklmnopqrstuvwxyz 1234567890

4. In 1984, Imagen Corporation released the first Lucida family of seriffed designs. They were simple in form and had several adaptations for lower resolutions, such as short serifs and wide inter-letter spacing to prevent collisions of character elements and clarity at small sizes and greater reading distances from screens, for which loose spacing is often helpful. We made the bold weight with twice the stroke weight of the normal weight, for unambiguous recognition of weight changes at low resolutions. Instead of a "just noticeable difference" we wanted a "dramatically noticeable difference".

These designs constituted the first new family of types for laser printing and screens. When we developed Lucida, most typefaces being digitized were adaptations of existing metal or photo faces.

The adaptations were sometimes made ethically, but often by plagiarism. We hoped that our development of new digital types would spur more original and more ethical approaches to type, and the Imagen firm, started by graduate students and AI researchers at Stanford, agreed. Another five years went by, however, before major firms began to produce original designs.

With rue my heart is laden

For golden friends I had,

For many a rose-lipt maiden

And many a lightfoot lad

By brooks too broad for leaping
The lightfoot boys are laid;
The rose-lipt girls are sleeping
In fields where roses fade.

A. E. Housman

5. In 1985, also with Imagen, we brought out a sans-serif companion to the original seriffed Lucida family. (The second stanza here shows the sans-serif variants, compared to the seriffed originals in the first stanza.) Many of the sans-serif Lucida features, including slightly darkish weight, generous spacing, big x-height, simple forms, and a humanistic style dating back to legible letters of the Italian Renaissance, made the family ideal for screen displays and user interfaces. Lucida Sans Unicode and Lucida Console have been bundled with Microsoft Windows since the early 1990s, and a version of Lucida Sans, named Lucida Grande (because it had a much bigger character set including Greek, Cyrillic, Hebrew, Arabic, Thai, and many signs and symbols) were adopted as the user interface fonts for Macintosh OS X from the beta tests of 2000 until 2014, when Lucida Grande was replaced by a digitized version of a more traditional "grotesque" sans-serif.



6. The individual letters of the first Lucida seriffed type were crafted in some unusual ways. Serifs and their brackets and some other features were polygonal

rather than curved. At the time of its design, 1983–1984, some printing and imaging systems used polygonal outlines, whereas other systems used curved contours, including circular arcs or cubic curves in Bezier or Hermite forms. We believed that it would be better to design the polygonal serifs ourselves rather than let some program automatically render them from curved forms. But, we kept curves for the larger aspects of the letters, like arches and bowls.

klmno

7. Here we see diagonal, straight, arched, and fully curved letters in combination, with the characteristic open spacing of the original Lucida.

klmno klmno klmno

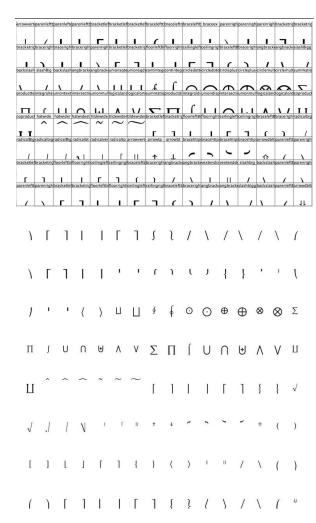
8. Comparison of the first Lucida (top) to Times Roman (middle), which appeared under its own name and in plagiarized forms under other names, to a modification of Lucida (bottom), fitted to the same widths as Times Roman and with modified serifs but keeping the x-height of Lucida (bottom). It looks condensed because the x-height is greater than that of Times Roman but the width is not. We later released a free version of this design under the name "Luxi Serif". It can still be found as a free font from web font firms and others. They are gratis but not open source, because we prohibited modification without our permission. We felt that the artistic aspects of the designs should be ours to control.

space	comma	period	slash	less	greater	A	В	С	D	Ε	F	G	Н	1	J
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К	L	М	N	0	Р	Q	R	S	T	U	v	W	X	Y	Z
K	L	Μ	N	0	P	Q	R	S	T	U	V	W	X	Y	Z
a	b	c	d	e	f	9	h	- 1	L.	k	1	m	n	0	р
а	b	С	d	e	f	g	h	i	j	k	I	m	n	0	p
q	r	5	t	u	v	w	×	У	z	mu	dotlessi	dotlessj	epsilon1	Gamma	Theta
q	r	S	t	и	ν	w	X	y	Z	μ	1	J	ϵ	Γ	Θ
Lambda	Xi	Pi	Sigma	Upsilon	Phi	Psi	alpha	beta	gamma	delta	epsilon	zeta	eta	theta	iota
Λ	Ξ	П	Σ	Y	Φ	Ψ	α	β	γ	δ	3	ζ	η	θ	ι
kappa	lambda	nu	xi	pi	rho	sigma1	sigma	tau	upsilon	phi	chi	psi	omega	theta1	phi1
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9. Lucida Math Italic, in the same character set as Computer Modern Math Italic. We began the design of math symbols for Lucida in 1985, in part because of the influence of Donald Knuth, who had invited me to join the Stanford faculty and to work with him. If there had been a guide to the various harmonizations needed for math fonts, it would have saved us a lot of effort and time, but there were none, except for examples from which some principles could be inferred. Also, we liked to figure out things by ourselves from first principles. We thought new technologies demanded new design ideas — a lesson we learned by studying with Hermann Zapf — so even had there been guides for math characters, we might have ignored them, being young.

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arrownort	arrowsou	arrowsout	arrowdbli	arrowdblu	arrowdblr	arrowdblo	arrowdblb	universal	existentia	emptyset	nabla	element	owner	coproduc	minus
minusplu	sasteriskm	radical	proportion	infinity	logicalane	~			integral		wreathpro	_	eguivasyn	equivalen	lessequal
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	circledivio	circledot	turnstilele	turnstileri	latticetop	perpendic	diamondr	ceilingleft	ceilingrig	floorleft	floorright	triangle	trianglein	openbulle	spade
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10. Lucida Math Symbol. We made the strokes of the math symbols fairly thick, in keeping with the general Lucida parameters, and we made many of the symbols fit in the standard figure widths, for easier composition of simple equations and usage in tabular composition, spreadsheets, etc. This tended to make the operators smaller than the letter symbols used for variables. We did this because we thought that the majority of uses of the Lucida math fonts would not be for traditional publishing of mathematics and science, but for simpler uses on screens.



11. We also produced a Lucida Math Extension font. In the first image, you can only see the tops of the characters, because to conform with Donald Knuth's approach with Computer Modern, many of the big delimiters and extensions "hang" from a central position in the character cell. The second image shows the full characters at a smaller size.

$$a = b$$
 $c \neq d$
 $p \geq q$ $v \div w$
 $a \otimes b$ $c \oplus d$
 $p \pm q$ $v \ominus w$
 $A \leq B$ $C \supseteq D$
 $P \supseteq Q$ $V \cup W$

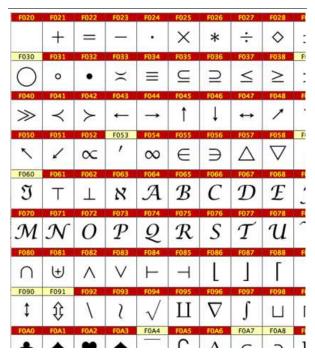
12. Above is the original Lucida math operators of various kinds with lower-case and capital letters. The weights are harmonized and the symbols are clear and resist digital degradation. We developed these math fonts in 1985, but they were not released until 1990 by Adobe. During that time, principally under the influence of PostScript, laser printing became an enabling technology for digital prepress for print publishing, and fonts intended only for lower-resolution printing were less favored. When the original Lucida and Lucida Math fonts were used for more traditionally printed math publishing, they seemed too strong and dark, although they held up well on screens and laser printers. As an editor at one academic press put it, "Lucida Math seemed too aggressively legible".

nn aa xx

13. For print publishing, and now for high resolution screen displays, we prefer legibility (at least when it is more subliminal than aggressive). In 1987, for Scientific American magazine), we made a modification of the original seriffed Lucida with increased contrast of thick to thin strokes, with more "modern" style serifs, and with tighter letterspacing. An editor at MacWorld magazine said it looked "brighter", so we named it Lucida Bright. It was used as the body text in Scientific American for nine years.

$$a = b$$
 $c \neq d$
 $p \geq q$ $v \div w$
 $a \otimes b$ $c \oplus d$
 $p \pm q$ $v \ominus w$
 $A \leq B$ $C \supseteq D$
 $P \supseteq Q$ $V \cup W$

14. For Lucida Bright, we developed "brightened" math characters. These were released by Microsoft in 1992 in TrueType font format. This prompted requests for TEX-adapted versions in PostScript format. At this point, we came to realize that the production of fully functional, TEX-compatible math fonts required more knowledge of TEX and more effort than we two designers could accomplish on our own.



15. Over the next few years, we worked with Berthold and Blenda Horn, of their firm Y&Y, at turning the Lucida Bright Math fonts into fully functioning Post-Script TEX math fonts. It was a tremendous effort made feasible only with the patient and extraordinary collaboration of the Horns.

space	plus	colon	equal	A	В	C	D	E	
	+	:	=	A	\mathcal{B}	C	\mathcal{D}	\mathcal{F}	5
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\mathcal{M}	\mathcal{N}	0	\mathcal{P}	Q	\mathcal{R}	S	$ \mathcal{T} $	u	1
bar	braceright	section	logicalnot	plusminus	paragraph	periodcen	multiply	divide	dag
1	}	§	\neg	±	1	•	×	÷	-
arrowup	arrowrigh	arrowdow	arrowboti	arrownort	arrownort	arrowsou	arrowsou	arrowdbll	arro
1	-	1	\leftrightarrow	1	1	1	1	=	1
ncremen	nabla	element	owner	product	coproduc	summatio	minus	minusplu	dot
Δ	∇	\in	\ni	Π	П	Σ	_	7	1
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16. During this development, we and Y&Y received requests for more math characters, so we kept adding them, and then came more requests for more characters. If we hadn't declared a halt and frozen the

fonts for release, we would probably be adding more characters still. (In truth, we still are, but now to the TUG releases, as we'll see.) To distinguish the Y&Y fonts from the previous Lucida Bright Math, which were in TrueType, not PostScript, and had fewer characters and different encodings, we named the Y&Y releases "Lucida New Math". I suppose we could have called them Lucida Bright Math 2.0, but that would have been confusing, too.

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224E	224F	2250	2251	2252	2253	2254	2255	2256	2257	1
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17. In the years since the Y&Y versions of the Lucida Bright New Math, the Unicode Consortium added math to the Unicode standard, with hundreds of characters beyond the standard TEX repertoire. Eventually, almost twenty years after the Y&Y Lucida versions, Karl Berry asked (following user requests) if we would like to work on OpenType versions in collaboration with TUG, and we began to work on OpenType versions of Lucida Bright and Lucida Math. The task turned out to be several times greater than the original Y&Y development, and also much greater than we anticipated. Partly because we revised some of our design notions during the development process, and partly because we added hundreds of characters.

$$a = b$$
 $c \neq d$
 $a = b$ $c \neq d$
 $a \otimes b$ $c \oplus d$
 $a \oplus b$ $c \otimes d$
 $A \leq B$ $C \supseteq D$
 $A \leq B$ $C \supseteq D$
 $P \supseteq Q$ $V \cup W$
 $P \supseteq Q$ $V \cup W$

18. In developing the OpenType versions, we decided to abandon our original notion that the math operators should match the figure widths; a comparison of the old and new operator sizes is shown above. Our experience over the years suggested that larger operators were more readable by mathematicians. This is anecdotal evidence; we did not conduct controlled laboratory tests of mathematicians reading equations, which should be done. Nevertheless, we made the math symbols bigger. (Interestingly, one of the most prolific legibility researchers of the 20th century, Miles Tinker, wrote his psychology Ph.D. thesis at Stanford in 1927 on "An Experimental study of legibility, perception, and eye movement in the reading of formulae" ... exactly a half-century before Donald Knuth began work on T_FX at Stanford.)

$$a = b$$
 $c+d$
 $a = b$ $c+d$
 $a = b$ $c+d$

19. We also made a series of bold weight versions of the math symbols. Unlike the original bold weight of Lucida, which had stems twice as thick as the normal weight, the Lucida Bright bold is more accurately a demibold, with stems 1.5 times as thick as the normal. Still very noticeable but not as domineering in text. (The above shows Lucida Bright Math OT, Lucida Bright Math Demi OT, and the original Lucida Bold.)

Not all these bold characters have well-defined semantics in mathematics, but we became accustomed to requests for such things, so we believed that if we designed the characters, mathematicians would find uses for them.

20. In our experience, and evidently in that of other type designers as well, the development of successful, original typefaces and fonts for mathematics requires technical collaboration. We wish to thank those who helped with the technical aspects of Lucida Math font development. For the Adobe PostScript versions of original Lucida Math: Daniel Mills. For the Y&Y PostScript versions, Lucida New Math: Berthold Horn and Blenda Horn. For the TUG OpenType versions, Lucida Math OT: Karl Berry, Khaled Hosny, and Michael Sharpe, plus a cast of testers, bug reporters, commenters, and other advisers: Barbara Beeton, Hans Hagen, Taco Hoekwater, Bogusław Jackowski, Mojca Miklavec, Norbert Preining, Will Robertson, Ulrik Vieth, Bruno Voisin.

ABCDEFG HIJKLMN OPQRSTU VWXYZ ABCDEFG HIJKLMN OPQRSTU VWXYZ

21. Work on the Lucida math fonts sometimes resulted in new stand-alone designs, especially scripts. The first such spin-off was "Lucida Calligraphy", a "brighter" version of the original calligraphic capitals in Lucida Math. Above, the original capitals are shown first, with the bright Lucida capitals below.

Lucida Calligraphy Normal

TEXT SAMPLES | CALLIGRAPHY NARROW NORMAL |

abcdefghíjkl

mnopqrstuv

wxyz

ABCDEFGHIJ

KLMNOPQR

STUVWXYZ&

0123456789

~!"@'#\$%'^**()

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22. Here is Lucida Calligraphy in the ASCII set. Originally distributed by Microsoft and now by other vendors, including Monotype. It is based on Italian Renaissance chancery cursive, but, like the other Lucida designs, adapted to digital rendering at a range of resolutions and technologies. We see it almost every day somewhere, often used in some surprising way.

Lucida Calligraphy Italic Art begins where geometry ends, and imparts to letters a character Art begins where geometry ends, and imparts to letters a character Art begins where geometry ends, and imparts to letters a characte Lite Art begins where geometry ends, and imparts to letters a characte Book Art begins where geometry ends, and imparts to letters a charact Text Art begins where geometry ends, and imparts to letters a charact Normal Art begins where geometry ends, and imparts to letters a charac Thick Art begins where geometry ends, and imparts to letters a charac ExtraThick Art begins where geometry ends, and imparts to letters a chara Dark Art begins where geometry ends, and imparts to letters a char ExtraDark Art begins where geometry ends, and imparts to letters a char Bold Art begins where geometry ends, and imparts to letters a ch Art begins where geometry ends, and imparts to letters a c Black Art begins where geometry ends, and imparts to letters a ExtraBlack Art begins where geometry ends, and imparts to letters

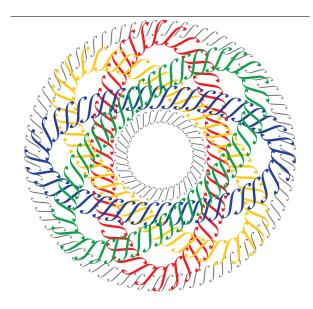
23. We have since expanded Lucida Calligraphy through a range of weights. Most of these are in ASCII only for English-speaking users. We are now developing additional characters sets for European languages and orthographies.

ABCDEFGHIJKLM NOPQRSTUVWXYZ abcdefghijklmnopgrstuvwxyz

1234567890 &@*?!

ABCDEFGHIJKLM NOPQŖŚTUVWXYZ abcdefghijklmnopgrstuvwxyz 1234567890 &@*?!

24. For the Lucida OpenType Math fonts, we have recently developed an English Roundhand script. It is not a copy of a standard traditional roundhand, but a style with more "zip" (equivalent to "élan") devised by Kris Holmes. This will soon be released in normal and bold weights, first through TUG. (The name hasn't yet been confirmed, but several dozen TUG members at the recent meeting in Toronto signed a petition requesting it be called "Typey McTypeface".)



The mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way.

G. H. Hardy

25. Designing so many hundreds, eventually thousands, of characters for mathematics, starting from scratch has been a thirty year adventure that is not over yet. The English mathematician G.H. Hardy once wrote what could serve as a guide for type design as well as mathematics. "The Mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas, like the colours or the words, must fit together in a harmonious way." (From A Mathematician's Apology.)





26. Kris Holmes could not be at the conference. but she contributed most of what makes the Lucida designs exciting as well as legible, so I want to close with a photo of her when she studied with Hermann Zapf many years ago, looking intently over Zapf's shoulder as he demonstrates how to write with colors; and then again with Zapf, a quarter-century later, when Kris was one of the judges for a Linotype Arabic type design competition.

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