#### Oh, oh, zero!

Charles Bigelow

#### Abstract

Despite exponential increases of computing power over the past half-century, at least one problem involving ones and zeroes has defied easy solution: how to shape the graphical forms of numeral '0' (zero) and capital letter 'O' (Oh) so a human reader can easily distinguish between them.

#### 1 Introduction

What follows is a look at three aspects of the zero-Oh problem.<sup>1</sup> First, a survey of computing and typographic literature discussing the problem in the 1960s and 1970s. Second, examples of practical solutions in digital fonts from the 1980s to present. Third, examples of the origins of the problem in the typography of the Italian and French Renaissance, and in English and American typography during the Industrial Revolution. The focus is on typographic symbols. For histories of mathematical notation before typography, see Cajori (1993) and Ifrah (1998).

#### 2 Zero versus Oh in computing

R.W. Bemer (1967), in a playfully entitled paper, "Toward Standards for Handwritten Zero and Oh: Much Ado about Nothing (and a Letter), or A Partial Dossier on Distinguishing Between Handwritten Zero and Oh" presents a compilation and discussion of proposals made between 1958 and 1966 to disambiguate the handwritten forms of zero and Oh. The goal of the study was to enable more accurate reading of handwritten code and data by the keypunch operators who typed punched cards for computer input.

It is doubtful that Bemer's paper led to lasting changes in handwriting, but Bemer also helped develop the American Standard Code for Information Interchange (ASCII), which, along with the advent of direct keyboard input, shifted the zero-Oh problem

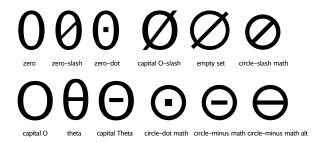


Figure 1: Zero and Oh: look-alike characters.

from handwriting reform, a fraught topic, to typographic legibility, which is no less problematic but substitutes mass-produced, prefabricated letters for the wayward penmanship of programmers. Not that this solved the problem of ambiguity in displayed or printed zeroes and Ohs, as exemplified by DIN 1450, the most recent legibility and typography standard from the German Institute for Standardization (DIN, 2013), which once again revisits the perennial problem of differentiating zero from Oh in contemporary typography.

Advances in font technology have complicated the problem by enabling fonts to contain much larger character sets, increasing the chances that several confusable letters and symbols may appear in a font or font family, especially in scientific and mathematical publishing. Figure 1 shows a set of characters similar to zero and capital Oh, from Lucida Sans and Lucida Math fonts.

The zero-Oh solutions proposed in Bemer (1967) include: a loop, flourish or stroke at the top of Oh; a slash through zero or Oh; a dot or dash in the center of Oh or zero; a rectangular shape for Oh but an elliptical shape for zero (or vice-versa); an Oh wider than zero; a lozenge orientation of Oh but square orientation of zero; a horizontal bar over Oh. One entry in the dossier briefly addresses the problem of differentiating numeral '1' (one) from capital letter 'I' (I), and numeral '2' (two) from capital letter 'Z' (Zee or Zed).

A subsequent tentative agreement on handwritten letter and numeral forms for computing was published in 1969 by an ANSI (American National Standards Institute) working group as "Proposed American National Standard: Presentation of Alphameric Characters for Information Processing" (Kerpelman, 1969). The proposal recommends a handwritten loop at the top of the capital Oh to distinguish it from a plain oval zero (Figure 2).

Kerpelman makes an intriguing observation regarding an evident difference in preference between two groups of programmers: "Programmers accustomed to use of business-type languages seemed to favor marking the zero. Those using mathematical

<sup>&</sup>lt;sup>1</sup> In this paper, the first reference to a character frames it in quotes, followed by the character's common name in parentheses. Subsequent references use the common name or a disambiguating term. Examples: 'O' (Oh) for capital letter Oh; '0' (zero) for numeral zero; '1' (one) for numeral one; 'l' (ell) for lowercase letter ell; 'I' (capital I) for capital letter I; 'Ø' (zero-slash) for slashed zero; '0' (zero-dot) for dotted zero. Characters representing numbers, e.g. 0123456789, are here called "numerals", although typographic literature typically uses the term "figure" for a character representing a numeral, e.g. "old-style figures", "lining figures" (Bringhurst, 1996). The Unicode character standard uses the term "digit" (Unicode, 2007). The Unicode standard distinguishes "character" as a unit of a writing system from "glyph" as a graphical mark representing a character, but that distinction is not fastidiously maintained in this paper.

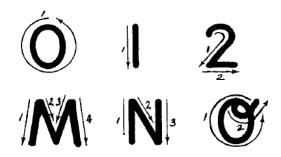
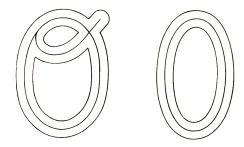


Figure 2: Zero and Oh from (Kerpelman, 1969).



**Figure 3**: Modification to Oh proposed by Vartabedian (1969, 1970).

or scientific-type languages conversely favored marking the letter." (Kerpelman, 1969). This preference difference, which can be characterized as between humanists and engineers, is a recurrent theme in the zero-Oh annals.

After these ACM and ANSI publications, the zero-Oh debate moved to a typographic forum, the Journal of Typographic Research, where psychologist Dirk Wendt (1969) analyzes the problem of discrimination and confusion between different forms of zero and Oh but does not recommend a single solution, other than an observation that zero narrower than Oh is more often interpreted correctly. In the same journal issue, a Bell Laboratories researcher, Allen G. Vartabedian, reports the results of a different legibility study and proposes that a loop or stroke be added to the top of the Oh to distinguish it from zero (Vartabedian, 1969). The proposal to add a loop to Oh is similar to that of Kerpelman.

In a letter in a later issue of the same journal, calligrapher and type designer Hermann Zapf (1970) objects to Vartabedian's proposal and proposes a contrary modification—the addition of a short horizontal stroke to the top right of the zero. Vartabedian (1970) responds with additional argument in favor of modifying the Oh. An engineer (Vartabedian) favors altering the Oh, while a humanist (Zapf) favors altering the zero, as shown in Figures 3 and 4.

# opqæœß&-+ 4567890A KLMNOPQ

**Figure 4**: Proposed modification to zero by Hermann Zapf. The zero resembles a tall lowercase sigma.

#### 2.1 Forms and ideas

Plato (or the character Socrates in Dialogues written by Plato) discusses how letters express ideas. He suggests, for example, that the letter omicron expresses roundness, though it is not clear whether Socrates (or Plato) is referring to the round shape of the letter or to the round shape of the lips when pronouncing the vowel signified by omicron. Perhaps both. Ancient Greek mathematics did not use a written symbol for the concept of nothing, but the atomist philosopher Democritus, possibly a contemporary of Socrates, uses the word "void" in contrast to "full", as attested by Aristotle in his Metaphysics (1989).

In modern semiotic discourse, the question can be asked: Is the graphical symbol "iconic"? Does the glyph resemble the thing it signifies? For most typographic glyphs, the answer is "no", but the zero glyph, a late addition to Latin script, is an elliptical or circular ring; its vacant interior containing nothing. Hence, it appears to be iconic. In writing and typography, an empty space separates symbols or groups of symbols, but does not signify something. Hence, to denote "nothing" there must be a mark that in some way delineates the presence of nothing. Yet, if the empty interior of the zero is iconic of nothing, then a mark inserted into it indicates that something is in the void, thus contradicting the iconicity of the empty glyph. In its long history, zero has sometimes been represented by a dot rather than a ring, so it could be argued that a zero-dot glyph is a double nothing, like a double negative is emphatically negative.

In set theory, an iconic representation of the empty set is a pair of braces framing an empty space:  $\{\}$ . The zero glyph has also been used to denote the empty set, but to disambiguate the number zero from empty set, glyphs made from zero with a slash (' $\emptyset$ ') or a circle with a slash (' $\emptyset$ ') have been adopted as symbols for the empty set. In Unicode, the empty





Figure 5: No Smoking symbols.

set character has code point hexadecimal 2205; Unicode does not separately encode zero-slash and circle-slash, instead considering them to be different visual forms. Both forms can still be provided as alternate glyphs within one font, as in the Lucida Math OpenType shown here. (To confound further, there is a slashed-zero variant appearance for zero itself which—though seldom used in seriffed fonts—often appears in sans-serif monospaced fonts (Figure 9), as we will discuss later.)

The empty set forms do not begin to exhaust the slashed circle symbols. A circle with a slash not projecting beyond the ring ( ${}^{\circ}$ O ${}^{\circ}$ ) has been adopted in European (and some American) signage to signify prohibition—"no" or "not". The prohibition slashed circle is usually, but not always, in an orientation opposite to that of the empty set, with the prohibition slash running from northwest to southeast but the empty set slash running northeast to southwest. Unlike the empty set symbol, the prohibition symbol usually contains something to be negated, such as a cigarette, as in Figure 5, which shows both orientations. The prohibition symbol is at code point hexadecimal 20E0 in Unicode.

Still more: the mathematical operator "circled division slash" ( $\oslash$ ) is oriented like the empty set but the slash does not protrude beyond the rim of the circle; it has code point hexadecimal 2298. And the programming language APL's "circle-backslash" character (' $\lozenge$ ') is encoded at hexadecimal 2349; it has various possible forms combining circle and backslash.

And, though not strictly circular, let us not forget the character Oh-slash 'Ø' (O with stroke, code point 00D8), and its lowercase form oh-slash 'Ø' (00F8), a common letter in the orthographies of the Scandinavian languages Danish, Norwegian, Faroese, and Sami.

Let's turn back to the common zero and Oh. In several recent fonts, excepting OCR-A and OCR-B from the 1960s and later fonts imitating them, the zero gets marked instead of the Oh. Hermann Zapf, however, who originally proposed to modify the zero with an additional stroke, found a calligraphic way to retain the purity of the empty, unadorned zero when he designed the Euler fonts for Donald Knuth and the American Mathematical Society in the early 1980s



Figure 6: Zero, Oh, oh, Theta, theta from the Euler typeface by Hermann Zapf.

(Figure 6). In the Euler roman typefaces (1987), Zapf drew the zero as a narrow elliptical shape with a calligraphic point at the top and a rounded base, almost as if it had been written with a pen in a single curved stroke. In contrast, the Euler Oh has a wider, smoother, almost super-elliptical shape. Hence, in the Euler typefaces, Zapf found a middle path for both engineers and humanists: neither zero nor Oh are marked by slashes, bars, dots, dashes, or gaps. Oh-like forms with interior marks represent traditional Greek letters, capital and lowercase theta ( $\Theta$  at hexadecimal 0398 and  $\theta$  at 03B8).

#### 2.2 Patterns of marking and legibility

Upon first impression, the varied proposals by mathematicians, engineers, psychologists, and designers seem to be in free variation. Some propose to modify the zero, others to modify the capital Oh; some want to add a diagonal slash, others to add a loop, others to add a dot, or a horizontal dash, or a projection. Some propose to reshape the curves of the zero, others to reshape the Oh, and at least one (Lo, 1967) suggests characters from another writing system, Chinese. Despite such variety, a few patterns can be discerned. One is that most of the proposals call for adding marks to existing forms, but none propose deleting parts of existing forms. Strokes and dots are to be added, but not gaps or breaks in contours.

The addition of black marks is in keeping with the common view of type forms, that the black marks are what are important, while the white spaces are not significant. Type designers, typographers, and graphic designers would say otherwise, but they are a small set of professionals, not the vast majority of readers. Another pattern is that the proposed marks are usually located at or above the midpoint of the character, and more often in the right upper sector than in the left. This follows a general tendency for Latin typographic alphabets to cluster most distinguishing features above the mid-point of the lowercase letter, near the x-line, and more often in the right upper sector than the left, a tendency noted by Huey (1908) and Legros and Grant (1916).

### ABCDEFGHIJKLM NOPQRSTUVWXYZ 0123456789 Xöü

Figure 7: FE-Schrift, designed by Karlgeorg Hoefer (with later modifications by others) for German vehicle registration plates. The gap in rectangular zero helps distinguish it from capital Oh; the serif arrangement of capital I helps distinguish it from numeral 1.

There is, however, at least one instance of an open gap to mark zero: on modern German vehicle registration plates, which use a font called FE-Schrift, originally designed by calligrapher and type designer Karlgeorg Hoefer (Figure 7). The zero is semi-rectangular and has a gap in the upper right corner, making it recognizably different from the Oh, which is an egg-shaped oval with unbroken contour. The numeral 1 is differentiated from the capital I in the size, orientation, and arrangement of serifs.

Zero vs. Oh confusion was uncommon before the computer era in part because letter versus numeral ambiguity was resolvable by context. A round, open form amidst numbers was presumably a zero. A round, open form among capital letters, or beginning a sentence or a proper noun, was presumably a capital Oh. In computing, however, symbol strings often mix alphabetic and numeric characters, thus rendering context insufficient as a means of distinguishing similar characters.

Among the proposals in Bemer (1967) is disambiguation of numeral one from capital I, though not from lowercase letter 'l' (ell). Few of the character sets used in computing in the 1950s and early 1960s included lowercase (American Standards Association, 1963), so there were fewer opportunities for confusion between numeral one and lowercase letter ell compared to those between zero and Oh. The numeral one vs. letter ell became more problematic when the 1966 revision of the ASCII character set added lowercase.

The graphical forms of numeral one and lowercase ell had been differentiated in traditional typography but were merged on typewriters, where a single glyph and key was used for both graphemes. Character encoding standards distinguished those characters numerically, e.g. in ASCII the numeral one is decimal 49 and lowercase ell is decimal 108, or in Unicode hexadecimal, they are 31 and 6C respectively, but as visual designs in fixed-width fonts, they have often remained similar in appearance.

### 1IIBOOD5SZ2

Futura (1927)

1IIBOOD5SZ2

**111B00D5SZ2** Frutiger (1976)

11BOOD5SZ2

1IIBOOD5SZ2

111BOOD5SZ2 I00
Lucida Grande & alternates (2001)

111B00D5SZ2 01 Frutiger Neue 1450 & alternates (2013)

### 1IIBOOD5SZ2 IIOO

**Figure 8**: Sans-serif typefaces showing: numeral one, capital I, ell; B, zero, Oh; D, 5, S; Z, 2. All types set at same body size.

#### 3 Zeroes and Ohs in contemporary fonts

Given the history of interest in the zero-Oh problem, and its transference to the realm of type design instead of handwriting, what solutions are found in contemporary typefaces? Many thousands of fonts are available today, but a small selection of widely used fonts can show the main features of the problems: see Figure 8.

In sans-serif typefaces, the problem of confusion between numeral one, capital I, and lowercase ell is more difficult than in seriffed faces because serifs function to distinguish capital I from lowercase ell and both from numeral one. (A seriffed capital I has four serifs, a lowercase ell three serifs with the upper left shaped differently than that of the capital I, and the numeral one has three serifs with the upper left serif distinguished in shape from that of both the I and ell.) In many sans-serif typefaces, capitals and lowercase ascenders are the same height, removing another distinguishing feature.

In the geometric sans-serif Futura by Paul Renner (1927), the zero is a narrow vertical ellipse and the capital Oh is wider and visually circular. The numeral one has a short horizontal stroke at upper left, and is the same height as capital I, while lowercase ell is noticeably taller than capital I or numeral one.

In the neo-grotesque sans-serif typeface Helvetica by Max Miedinger and Eduard Hoffman (1957), the zero is distinctly narrower and slightly shorter than the capital Oh. The numeral one is distinguished from capital I and lowercase ell by a ramplike stroke at upper left. The lowercase ell and capital I are the same height and differentiated only by a slightly greater weight of the capital I—a difference that is nearly or entirely imperceptible at small sizes and low digital resolutions.

In the Transitional-style sans-serif Frutiger by Adrian Frutiger (1976), the zero is noticeably narrower than the capital Oh, but both are the same height. The lowercase ell is slightly taller than the capital I, and the numeral one is differentiated from both capital I and ell by a short diagonal stroke at upper left.

In the humanist sans-serif Lucida (1985) by Charles Bigelow and Kris Holmes, the zero and Oh are differentiated by width but not height, reflecting the study by Wendt (1969). The lowercase ell is noticeably taller than the capital I (except at very small sizes and low resolution) and the numeral one is differentiated from both by the short diagonal stroke at upper left.

In Verdana by Matthew Carter (1996), the zero and Oh are differentiated by width but not height; baseline serifs are added to the numeral one and four serifs to the capital I for greater differentiation of the three characters.

In Lucida Grande, based on Lucida Sans, the numeral one is reworked with baseline serifs but the default zero and Oh are the same as in the original version. The Lucida Grande font also includes both slashed and dotted versions of zero, as well as a seriffed variant of capital I, but these are not the default forms.

In Neue Frutiger 1450, by Adrian Frutiger and Akira Kobayashi (2013), the capital I acquires four serifs, the zero a dot, and the lowercase ell a curved exit similar to that of lowercase 't'. An open zero and rectangular lowercase l are provided as alternates.

In Lucida Grande 1450 by Bigelow & Holmes (2013), the slash zero and seriffed capital I are defaults, along with a lowercase ell with exit stroke. The numeral one has baseline serifs as in standard Lucida Grande. A dotted zero and open zero, as well

as rectangular lowercase ell and rectangular capital I are provided as alternates.

From this look at recent and widely used fonts, we can see that from the many proposals for character disambiguation made over the past 50 years, a few trends have emerged and converged. In the zero vs. Oh pair, the zero is almost always the character that receives an added element, usually an internal diagonal slash or an internal dot. The Oh does not get decorated with loops or twiddles, despite such suggestions in Bemer (1967) and Vartabedian (1969). In proportionally spaced fonts, the zero form is generally a narrow ellipse, while the capital Oh is usually a broader, nearly circular form, reflecting the findings of Wendt (1969). In terms of the debate between humanists and engineers, the 'humanist' side has won—the numeral gets modified, not the letter.

In the case of numeral one versus capital I and lowercase ell, the outcome is more like a draw. The numeral one is given baseline serifs in several sansserif typefaces, while the capital I is given serifs in others, and in some sans-serif fonts, seriffed versions of both characters occur. The lowercase ell is variable, sometimes differentiated from capital I and one by a serif or a stroke in upper left or lower right, or both.

These trends are seen to an even greater degree in monospaced fonts (Figure 9), which have the added constraint that zero cannot be differentiated from Oh by width because all characters in a monospaced font must of course have the same advance width. Although typewriters as machines have become obsolete, monospaced fonts developed for typewriters, despite their retro appearance and association with old technology, are flourishing in the digital era. Several new monospaced font families have been designed since the widespread adoption of digital font technology for laser printing and computer displays in the 1980s. Just as the forms of letters made the leap from handwriting to print in the 15th century, they have made the leap from analog to digital technology in the 20th and 21st centuries. Technology has influenced the forms of letters over the centuries, but the visual forms themselves can exist beyond any particular technology, reminding us of Plato's philosophy of eternal forms.

Courier by Howard Kettler (1955) is a seriffed monospaced font for IBM typewriters. It became the most used typewriter font of all time and was therefore one of the first to be implemented in digital form. A seriffed font, Courier has two baseline serifs on numeral one, three serifs on lowercase ell, and four serifs on capital I, all as would be expected of a seriffed typeface. Numeral zero is differentiated from capital Oh by height, because in Courier, unusually, 111B00D5SZ2

1IIBOOD5SZ2Letter Gothic (1962

111B00D5SZ2 Lucida Sans Typewriter (1986)

111B00D5SZ2

111BOOD5SZ2 0 Lucida Console (1993) & alternate (2013)

111B00D5SZ2 Andale Mono (1997)

111B00D5SZ2 00ll

111B00D5SZ2 Inconsolata (2009)

111B00D5SZ2

**Figure 9**: Monospaced typefaces showing: numeral one, capital I, ell; B, zero, Oh; D, 5, S; Z, 2. All types set at same body size.

the numerals are taller than the capitals and also more loosely spaced; thus zero is a tall, narrow ellipse while capital Oh is shorter and more nearly circular. Capital I is noticeably shorter than either numeral one or lowercase ell.

Letter Gothic by Roger Roberson (1962) is a "fineline" sans-serif for IBM Selectric typewriters. The numerals and capitals are the same height. Numeral one has two base serifs and diagonal upper left stroke, capital I has four serifs, and lowercase ell has a single horizontal serif at upper left. The zero and capital Oh are indistinguishable. (Letter Gothic looks lighter than the other fonts because the original design had a light stroke weight to compensate for ribbon spread in typewriting.)

Lucida Sans Typewriter (1986) by Bigelow & Holmes distinguishes numeral one, capital I, and lowercase ell by position and number of serifs: one upper left serif on ell, two serifs plus upper diagonal stroke on numeral one, and four serifs on capital I. The numeral one is distinguished from the lowercase ell by the shape of the stroke or serif at upper left, diagonal on numeral one and horizontal on ell, by the presence of baseline serifs on numeral one but not on ell, and slightly greater height of ell and other ascending lowercase characters compared to numerals and capitals. Zero is narrower than the capital Oh but has the same height and does not have other distinguishing marking.

Monaco, by Bigelow & Holmes for Apple (1991), derived from bitmap fonts by Susan Kare (1984), has capitals and numerals of equal height but slightly shorter than ascenders. Capital I has serifs, numeral one has baseline serifs as well as the diagonal stroke in the northwest, and lowercase ell has serifs only at upper left and lower right. Zero has a diagonal slash, but is not distinguished from capital Oh by height or width.

Lucida Console (1993) by Bigelow & Holmes has capitals noticeably shorter than the numerals, due to technical constraints in Microsoft Windows NT, for which the font was first developed. Most of the other letters and numerals are similar to those in Lucida Sans Typewriter. A slashed zero was considered, but the designers and Microsoft decided that the height difference would be sufficient to distinguish the two characters. However, in a new version to be released in 2013, B&H have added the slash to the zero.

Andale Mono by Steve Matteson (1997) has serif patterning similar to that of Monaco for the figure one, capital I, and ell. The zero is dotted, rather than slashed, and slightly narrower than capital Oh but the same height. Capitals, numerals, and lowercase ascenders are equal in height.

Consolas by Lucas de Groot (2006) has a slashed zero as default, but as an OpenType font, it includes an alternate dotted zero and open zero. All the zeroes are slightly narrower than capital Oh but the same height. The font also contains old-style numerals including all three zeroes, which align with the lowercase. The numerals and capitals are equal in height and shorter than lowercase ascenders.

Inconsolata by Raph Levien (2009) has a slashed zero narrower than, but the same height as, the capital Oh. The numeral one has the usual diagonal stroke at upper left but lacks baseline serifs, thus being differentiated from lowercase ell, which has three serifs, as in Consolas.

Lucida Retro by Bigelow & Holmes (2013) has a slashed zero narrower than the capital Oh and a two-seriffed lowercase ell as in Andale and Monaco. Proportions and heights are similar to those of Lucida Sans Typewriter, from which it is derived. Additional differences occur in certain other lowercase letters and symbols.

Thus, as we can see, in most digital monospaced sans-serif fonts strict modernist design purity is sub-ordinated to legibility, because many of these fonts are used in programming and terminal and console windows of operating systems and programming environments, where legibility is paramount. Despite the fonts being sans-serif, letters subject to confusion, like capital I and lowercase ell, are given serifs to distinguish them from each other and from numeral one, which may also be given baseline serifs, contrary to sans-serif purism.

If there is a lesson to be drawn from these comparisons, it is that in the computer era, the trend has been toward more marked differentiation of confusable forms, even when the markings are contrary to historical tradition or design purity. It appears that the particular details of individual character designs are not as important as the overall structure of the set of differentiations. Zero may be distinguished from capital Oh and from lowercase oh by slashes, dots, heights, or shapes.

The numeral one, lowercase ell and capital I may be distinguished by the presence, location, and orientation of serifs, but the exact number and location of serifs may vary depending on the preferences of the designers or functions of the font.

#### 4 A brief historical survey

Implicit in Bemer (1967) is an assumption that zero—Oh confusion results from lack of clarity in the handwriting of the 1950s, when computing began to be used widely in industry, government, and academia. The graphical problem long predates computing, however, and may be traced back to the handwriting and early typography of the Italian Renaissance, when our modern alphabets took form and when the Arabic numerals began to be integrated into humanistic and scientific writing and publishing.

Our modern roman and italic typefaces are derived from humanist handwriting, which amalgamated two distinct forms of the Latin alphabet: Roman capitals, which had reached their canonical forms by the 1st century A.D., and Carolingian minuscules, which were based on cursive descendants of the capitals re-formalized by scribes in the court of Charlemagne around the end of the 8th century A.D. Most of the minuscules, which in typography

12 3 + 4 4 6 6 1 8 9 1 10 11 12 13 12 14 16 17

18 19 20 21 22 23 24 25 26 21 28 29 30 31

32 33 34 35 36 37 38 39 40 81 82 83 84 84 85 86

61 62 63 64 64 66 67 68 69 70 71 12 73 72 74

76 71 18 19 80 81 82 83 86 81 86 81 88 29 90 91

92 93 94 94 95 94 98 99 100 2000 190 3004 200 190

Figure 10: Numerals in gothic handwriting, 1459. http://commons.wikimedia.org/wiki/File: Ms.Thott.290.2%C2%BA\_150v.jpg

are commonly called "lowercase", evolved into forms different from their capital antecedents, such as 'a' from 'A', 'b' from 'B', and 'e' from 'E', but minuscule 'o' retained the form of capital 'O'. Capital and minuscule Latin letterforms evolved in isolation from the Hindu-Arabic numerals, which were not introduced to Europe until the end of the 10th century A.D. The zero appears not to have been used in Europe to any appreciable extent until early in the 13th century Ifrah, 1998). The influential mathematical book Liber Abaci, written around 1202 by Leonardo of Pisa (known as 'Fibonacci'), brought Arabic numerals and zero into wider use in bookkeeping and mathematics, but roman numerals, based on letters and letter-like forms, continued in wide usage.

Although the glyphs of writing systems change form over time, the changes generally maintain differentiation between the elements of the system. The centuries of evolution of Latin letters separately from Hindu-Arabic numerals meant that there was no pressure to distinguish the numeral zero from the letters Oh or oh, because they were parts of different systems. From the 9th to the 15th century, Carolingian minuscules gradually morphed into the various gothic hands known as blackletters or broken scripts. As Arabic numerals increased in usage, they were used with gothic handwriting, in which the zero glyph tends to have a slight point where the loop stroke joins itself (Figure 10).

#### 4.1 A sampling of early zeroes in print

In both gothic and humanist manuscripts, Arabic numerals had ascending and descending strokes, what we call "old style" numerals today. The '0', '1' and '2' were roughly x-height; the '3', '4', '5' and '9' descended below the baseline, and the '6' and '8' ascended above the baseline. These features were continued in typographic fonts. The symbol zero in Renaissance handwriting and typography was usually

a circular shape roughly like lowercase oh. Occasionally, the zero was more pointed when used with gothic scripts, and more circular when used with humanist scripts. Following are some specific examples.

1473. A calendar by the German mathematician called "Regiomontanus" (Johann Müller von Königsberg; see Figure 11) was printed in Nuremberg in 1473 using Arabic numerals including zero. An example reproduced by Cajori (1993) is crude, and the forms of the numerals, including zero and oh, are more variable than would be expected from movable type, but the 1474 edition shows a nearly circular zero glyph without strong thick-thin shading, while the oh glyph is larger and darker with more contrast of thick and thin. Also noteworthy is the use of a humanist roman typeface in a book printed in Germany at this relatively early date. Thus, zero was distinguished from oh early in printed books.

The 1476 edition printed by Erhardt Ratdolt in Venice also uses the circular zero in movable type, with a humanist roman typeface. Ratdolt's edition has the distinction of the first known ornamented title page in a printed book, as well as extensive (though not the first) use of rubrication in print.

1474. The Fasciculus Temporum, an encyclopedia of history, by Werner Rollevinck, printed by Arnold ther Hoernen in Cologne in 1474, makes extensive use of Arabic numerals, cut in gothic style, including zero, in dating events in history (Figure 12 shows a sample from a different edition).

1478. An arithmetic text, Arte dell'Abbaco (author unknown), written in Venetian dialect and printed in Treviso in 1478 by Gerardus de Las de Flandria or Michele Manzolo shows a roughly circular zero glyph that is apparently the same as the lowercase letter oh glyph. A peculiar twist is that the glyph for numeral one has a dot above it and it appears identical to the lowercase letter 'i'. Apparently, neither author nor printer deemed it necessary to distinguish those numerals from their similar letters in this humble text. See http://www.columbia.edu/cu/lweb/eresources/exhibitions/treasures/html/160.html.

1491. According to Ifrah (1998), the word "zero" first appeared in print in De Arithmetica Opusculum, by Philippi Calandri, printed at Florence in 1491. The Arabic name for zero, "sifr" meaning "empty, void", was borrowed into medieval Latin as "zephirum" in Fibonacci's work, and later simplified to Italian "zefiro" and then shortened to "zero"). German "Ziffer", French "chiffre", and Spanish "cifra", which include all numerals, come from the same Arabic word, as does English "cipher", which can mean zero, or more generally a numeral,

TIME		un					
K1:	sh.	htt	lod	5	6	5	6
17 WWW	Ytero Jac	20	3	0	11	0	13
	der adject-B-Steffans	21	4	0	26	0	26
	der achtet 4 Johanns	22			10	1	2
402110	der adjet der Emblein	23	٨	1	27	1	33
4 c Ron		24	8	2	6	2	4
6 F 8109	Opare.	24	9	Z	19	2	18
VH V19		26	11	3	2	3	1
8 6409	5 Ethant billhous	21	12	3	16	3	14
9 6 4 209	S Tulian und fem gefelle	28	13	3	29	3	28
	spaul amfidel	29	19	9	12	4	11

JANVARIVS.	OLIS LVNAE Afende	mb
	APRI SGISGIS III	ijda
r A Circucifio domini	20 3 0 13 0 13 1 24	8
z b a no Octaua.s. Stephani	21 2 0 26 0 26 1 24	10
3 c 3 no Octaua.s. Ioannis	22 6 1 10 1 9 1 302	11
g d z no Octaua.s. Innocentum	23 A  1  23   1  22 1   436	13
4 e Non Vigi	ia   2 8   2 6   2 4 1   40	18
6 f 8 id? Epiphania domini	29 9 2 19 2 18 1 17	16
A g Aid) Clauis . lxx		18
8 A 6 id? Erbardi episcopi	ZN 12   3   14   3   14   7 z	109
9 b 4id? Iuliani & sociorum eius	28   13   3   29   3   28   1   56	20
10 c   2 id?   Pauli primi eremite	29 12 2 12 2 11 2 0 2	ZZ

		TITI	.IANVARIVS.	SOL	IS	I	VI	VA	E
		KY	1400   1.00 P. HOO F.	CAPI	a.	S.	G.	S.	G.
1	A	IVL	Circucifio domini	20	3	0	13	0	13
2	Ъ	4 noñ	Octaua.s.Stephani	21	4	0	26	0	26
3	c	3 noñ	Octaua.s.Ioannis	22	6	-1	10	1	9
4	d	2 noñ	Octaua lanctoru Innocentu	23	7	1	23	1	22
5	e	Non	Vigilia	24	8	2	6	2	5
6	f	8 id9	Epiphanie domini	25	9	2	19	2	18
7	8	7 id9	Clauis.lxx.	26	11	3	2	3	1
8	A	6 id9	Erbardi episcopi	27	12	3	15	3	15
9	Ь	s id9	Iuliani & fociorum eius	28	13	3	29	3	28
10	c	4 109	Pauli primi eremite	29	14	4-	12	4	11

Figure 11: Three different editions of the calendarium of Regiomontanus: (a) 1473 Nuremberg (from Cajori (1993), p. 97); (b) 1474 Nuremberg (http://daten.digitale-sammlungen.de/0003/bsb00031144/images/index.html?fip=193.174.98.30&id=00031144&seite=7); (c) 1476 Venice (from the Digital Rare Book Collection at the Vienna University Observatory, http://www.univie.ac.at/hwastro).

or a secret (code). See http://www.metmuseum.org/toah/works-of-art/19.24 and http://www.lib.umn.edu/apps/bell/map/PTO/GEO/clklg.html.

1494. In Luca Pacioli's Summa de Arithmetica (Somma di arithmetica in Italian) printed by Paganinus de Paganinis in Venice in 1494, the zero is nearly circular without any contrast of thick to thin strokes, whereas in the gothic rotunda text face of the book, the letter oh is taller, more pointed, and compressed. The difference is evident whether the zero appears in tables with other numerals or in linear text with numerals and letters. A high resolution digitized example can be seen on-line from the Max Planck Institute for the History of Science. (It is

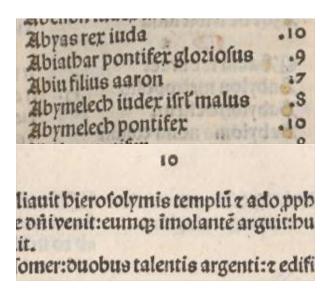


Figure 12: Zeroes and ohs from Fasciculus Temporum 1485, Venice: (a) Name index (A's); (b) Folio (10). http://archive.org/details/OEXV552P1

clear enough that an arithmetically minded reader should be able to spot a numero-typographical error in one of the multiplication examples on (digital) page 77, book folio 31.) The printed impressions of the letters and numerals are somewhat variable due to the textured, hand-made paper and imperfections of early printing, but the zeroes in the tables noticeably differ from the lowercase letter oh in the text columns. See http://echo.mpiwg-berlin.mpg.de/ECHOdocuView?url=/permanent/

archimedes\_repository/large/pacio\_ summa\_504\_it\_1494/index.meta&start=71&pn=77.

1498. A manuscript of another work on mathematics by Luca Pacioli, De Divina Proportione (Figure 13), written in Milan in 1494, is illustrated with drawings of polyhedra attributed to Leonardo da Vinci, but, alas, the scribe who wrote the text in an elegant humanist bookhand remains anonymous. The text uses Arabic numerals but the scribe, despite evident mastery of the edged pen, does not appreciably distinguish zero from oh. This is probably because it is difficult to write an unshaded circular form with an edged pen. Although Arabic numerals are used in the diagrams and calculations, the page numbers are in roman numerals, indicating the conservative power of the older system.

In books printed in humanist typefaces (our "roman" style) before the 16th century, Arabic numerals appear to have been rare in body texts, though appearing in indices, lists, tables, and calculations. Even page numbers or "folii" were usually printed in roman numerals until the 16th century. My impression of the paucity of the Arabic numerals is,

ong. Sienno doi lince così dittise bedittisa in c. clastia magiori. Elaltra de clastia magior partommo diciamo dequeste doi amo de infinite altre Lequali è possano per uia de Arithme le ponendo ab. 10. ac. seria Raltra 15 m R 125. Eponendo de. Ri 80 m 6. clastra seria 18 m

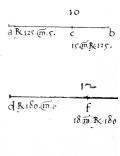


Figure 13: From De Divina Proportione by Luca Pacioli, manuscript dated 1498, scribe unknown. Aboca Edizione, Italy, 2010. Facsimile of Manuscript 210, Library of the University of Geneva.

however, based on a fragmentary survey, so further investigation might alter our understanding.

The reason for such rarity of Arabic numerals in humanist texts is unclear, but may have been because the humanists were interested in classical philosophical, literary, and historical works more than mathematical and practical treatises on arithmetic and accounting. Humanist artists and architects did produce treatises on the design of roman capital letters by Euclidean constructions, but these did not extend to minuscules (lowercase) or numerals. It appears that humanists were more interested in Oh than zero. Likewise, in De Divina Proportione, Pacioli also constructed the roman capitals, but not minuscules or numerals.

The ring-shaped zero, rather than a calligraphic zero, begins to appear with humanist roman type-faces in the last decade of the 15th century. In humanist works, context was probably sufficient to differentiate numeral zero from letter oh in most instances, but our next example shows one case of possible confusion, perhaps due to the compositor or to the absence of the characters in the roman font. (When setting type by hand, it is easy to confuse look-alike letters, such as 'p' and 'q' (hence the maxim to mind them) and possibly zero and oh.)

1498. Aldus Manutius in Venice published the Opera of Angelus Politanus in 1498 (Figure 14). On a page of "Epigrammatum graecorum", the Arabic numerals (such as "1490") use a zero that is the wrong size and alignment for the rest of the numerals. Instead, the zero looks to be the same size and alignment as the oh of the roman text font, Aldus' first roman, cut by Francesco Griffo. The type is approximately 15 point in size, but the numerals seem somewhat smaller. Possibly, the compositor confused letter oh with numeral zero, or perhaps the set of numerals didn't include a zero, though that

### Τυ λάγμοι μαλακώς οωτορ διών εμπνε έρωτας ακὶ γλώ Ταν γλώ Τα και πρι κλεκτον έχει 1490. Latinum nescio cuius Romæa Ciampolini gracum deine

Figure 14: Politanus, Opera, printed by Aldus Manutius, 1498. Munich Digitization Center and Digital Library. http://daten.digitale-sammlungen.de/0005/bsb00050563/images/index.html?fip=193.174.98.30&id=00050563&seite=903

C ch.3.f.1.1.16.cotemto pro cotempto 1.20.fuspicare.p trare.p parlare.ch.5.f.1.1.0. fa parturisce.ch.6.f.1.1.10. Gratis ma.seguitoe.p seguiroe.ch.6.f.1.14.seruli pro ferali. Q mătime.ch.3.f.5.1.2. Garo.pro Ciaro. No senu Venetiis Mense decembri.M.ID.inædibus Aldi Manuti

**Figure 15**: Error page of Hypnerotomachia Poliphili (author unknown) printed by Aldus Manutius, Venice, 1499. Rochester Institute of Technology, Cary Collection.

seems less likely, because Griffo, or whoever cut the numerals, should have been able to cut a zero as well as the other numerals.

1499. A year later, Aldus did use a zero properly aligned with other numerals cut at a very small size, on the "errata" page of the Hypnerotomachia Poliphili printed in 1499 (Figure 15).

The Hypnerotomachia is composed in a large (approximately 15 point) humanist roman typeface cut by Griffo. The lowercase is based on the earlier roman, but the capitals are new. Numerals in the main body of the book are roman numerals, but in the "erratta" page at the end, small Arabic numerals (approximately 60% of the x-height of the text face) are interposed. These numerals have the ring-shaped zero design that became standard for roman faces in the 16th century. See "20" in line 1 and "10" in line 2.

These examples suggest that Italian Renaissance readers of humanist manuscripts and printed books would have been unlikely to confuse zero with capital Oh; confusion could have occurred between zero and lowercase oh. In most instances, however, numerals and letters occurred in different contexts, which would have lessened the chances of confusion. Where they did co-occur, numeral zero and lowercase oh were differentiated by different size and/or different ductus, at least in type, where the more circular form and lack of thick-thin shading in the zero distinguished it from the humanist oh, which did have thick-thin shading.

Later, in the 16th century and especially in France, Arabic numerals gradually became more often used with roman typefaces. A type specimen

### Æ.A.B.C.D.E.F.G X.Y.Z. q.a.b.c.d.e.f.g.h.i ū.ff.st.si.fi.fl.st.e.æ.æ. 1.2.3.4.5.6.7.8.6.0 Non niverbo quodegreditur?

Figure 16: Type specimen of François Guyot, circa 1565. Type Specimen Facsimiles, ed. John Dreyfus. Bowes & Bowes and Putnam, London. 1963. Original document in Folger Library, Washington, D.C. Note numero-typographical error of '6' substituted for rotationally symmetrical '9'.

Romaine, de Rob, GranIon nommécela Gaillarde. en Anuers

Oratio Susannæ. Danielis xiij. Deus æterne, qui absconditorum cognitor

Figure 17: Gaillarde by Robert Granjon, 1570. Type Specimen Facsimiles II, H.D.L. Vervliet and Harry Carter, ed. John Dreyfus. Bodley Head, London, and University of Toronto Press, Toronto. 1972. Original document in Plantin-Moretus Museum, Antwerp. Original approximately 9 point.

attributed to François Guyot (Figure 16), circa 1565, displays complete Arabic numerals for several sizes of type. As in 15th century Arabic numerals, Guyot's numerals had ascending and descending forms. The zero is cut as a small circular ring roughly the size of a lowercase oh but without thick-thin shading. Guyot's types are cut in the style of Garamond, a canonical form in 16th century typography, and may be the earliest example of Arabic numerals cut for each size and style of type by the punch-cutter and cast by the typefounder.

A specimen of a small (approximately 9 point) roman, named "Gaillarde", cut by Robert Granjon, is dated 1570, with the circular ring-form zero (Figure 17). (The cutting is very fine, but the photo reproduction of the printed specimen makes it look rougher than it is.)

Thus, during the second half of the 16th century, Arabic numerals became incorporated into common expectations of what characters a "font" contained—at least capital and lowercase letters, punctuation,

0123456789 00 0123456789 00

OI23456789 OO 0123456789 OO Adobe Garamond

OI23456789 OO 0123456789 OO Galliard

Figure 18: Old-style and lining numerals from three modern revivals of Oldstyle typefaces: Sabon (Garamond), Adobe Garamond, and Galliard. Old-style numerals are in the first line, with old-style zero and lowercase oh for comparison, and lining numerals in the second line, with lining numeral zero and capital Oh for comparison.

and Arabic numerals. This amalgamation of disparate forms became the standard in printing from the 16th to the 19th century.

Modern revivals of 16th to 18th century types often include old-style numerals in addition to lining numerals, which are usually capital height (Figure 18).

Sabon by Jan Tschichold (1967) and Sabon Next by Tschichold and Jean François Porchez (2002) are revivals of types cut by Claude Garamond circa 1550 (an exact date is difficult to ascribe because Tschichold may have used more than one Garamond model). Porchez suggests that Tschichold's design was also influenced by types cut in Garamond's style by Guillaume Le Be, a younger contemporary of Garamond. In Sabon, the old-style zero is shaded but with heavier strokes at top and bottom instead of left and right, thus reversing traditional shading so as to reduce potential confusion between zero and lowercase oh. This zero design may be an invention by Tschichold, not Garamond. In the lining numerals of Sabon, the zero is capital height and distinctly taller than lowercase oh, so there is little possibility of confusion with oh. The lining zero has traditional shading—lighter strokes at top and bottom, heavier at left and right, as with traditional capitals, which reduces the difference between zero and capital Oh, but because the zero is distinctly narrower than capital Oh, the difference between them is evident.

Adobe Garamond by Robert Slimbach (1989), another revival of types cut by Garamond, uses a monoline ring form of zero like that seen in the Guyot specimen.

Galliard by Matthew Carter (1978), based on designs by Robert Granjon, also uses the monoline ring zero. The name is taken from Granjon's "Gaillard" type but is not an exact copy of that particular size.

In the late 19th century, the typewriter was developed and became popular. Huckleberry Finn, published in 1884, is said to have been the first typewritten manuscript submitted to a printer. The dominant style of printing type was known as "Modern", which includes a broad range of designs, from the elegant cuttings of Bodoni and Didot, used today for high-fashion advertising, to their workaday descendants, including Scotch Roman, American Monotype Modern 8a and Donald Knuth's Metafont derivative of it, Computer Modern.

Modern typefaces arose in the last decades of the 18th century, and included new proportions for the designs of numerals. Instead of the old-style numerals with ascenders and descenders extruding above the x-line or below the baseline, Modern-style numerals were cut so that the tops of the numerals all aligned. The first of these equal-height numeral sets was a late Transitional face cut by Richard Austin for John Bell, in 1788; the numeral height was intermediate between capital height and x-height. This style of numeral was adopted by other English and Scottish type foundries. The typeface called Scotch Roman is derived from types originally founded in the early 19th century by Scottish foundries, in particular the William Miller foundry in Edinburgh, Scotland. Certain of the Scottish types were recast nearer the end of the 19th century and sold in the U.S. under the name Scotch Roman. The dark version of Scotch Roman produced in the early 20th century by Monotype has lining numerals slightly shorter than the capitals.

The 1815 catalog of the London typefoundry of Vincent Figgins shows lining numerals for a range of text faces, and the numeral height is equal to that of the capitals. The 1828 catalog of the Edmund Fry typefoundry also shows lining numerals for a range of text faces, and the numeral height is equal to that of the capitals.

Bringhurst (1996) suggests that this change from old-style to lining numeral designs derived from hand-written numerals in English shop signs and placards in the 18th century, and was thus a consequence of the rising British middle-class. However that may be, it is evident that by the time the typewriter was developed at the end of the 19th century, the

## 0123456789 000

## **0123456789 0O**o

Scotch Roman

**Figure 19**: Revivals by Monotype of the Transitional typefaces Bell and Scotch Roman. Numerals align in height, but are slightly shorter than the capitals, as seen in the zero Oh oh combinations.

standard model for numerals had become the Modern style of equal heights (Figure 19).

To reduce complexity, both in the mechanics of the machine and in the mind of the typist, nearly all typewriters used monospaced (fixed-width) characters. (An exception was the IBM electric Executive typewriter in the 1940s and 1950s, and some models of the IBM Selectric typewriters made in later years, which used proportional spacing.) In the "fonts" of most typewriters, therefore, zero and Oh could not be differentiated by width. Moreover, most typewriter faces were unshaded or mono-line, that is, lacking the thick-thin modulation of printing types, so zero and Oh could not be differentiated by stroke shading. Thus, a zero and an Oh with same width, height, and monoline stroke looked very much the same in typewriting.

In typewritten documents before the computer era in mid-20th century, context was presumably sufficient to distinguish numerals from letters, but in computing, there appears to have been greater mingling of numerals with letters, giving rise to the issues discussed by Bemer (1967). In computer printing, the symbol shapes were occasionally differentiated by making one more rectangular and the other more oval, but there was no agreement on which symbol should be which shape, so confusion continued. Exacerbating the problem, similar confusion of zero with Oh also occurred in speech, when the numeral zero was, and indeed still is, often pronounced as "oh". A comment in Berner (1967) points out that the popular Boeing 707 aircraft was called a 'sevenoh-seven', not a 'seven-zero-seven'. On a telephone dial or keypad, the "Oh" for "Operator" is the zero key, not the 6 key (covering MNO). Telephone area codes are likewise spoken with the vowel oh or the spelling "oh" representing zero, as in 'five-oh-three' (503), the area code for northwest Oregon.

On many manual typewriters, the numeral one and lowercase letter ell were merged entirely, with one glyph on one key representing both graphemes, presumably for keyboard economy. In computing, the one-ell pair became visually confusable when low-

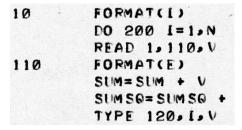


Figure 20: Part of a Fortran program printed by an ASR 33 Teletype. In this capitals-only font, the zero has a slash to distinguish it from the Oh. The sample shows that worn or poorly adjusted print heads may erode the differences between characters, such as numeral one and capital I in the expression "I=1". http://www.pdp8.net/asr33/pics/fortran\_printout.shtml

ercase was added to the ASCII standard (USA, 1967). As in the case of zero and Oh, context was not sufficient to differentiate such glyphs in the computer era.

When computer input and print-out began to be done through Teletype machines, the zero and Oh were distinguished by a slash through the zero, as in the print-out from a Teletype ASR 33 (Figure 20).

Although traditional typewriter fonts often did not distinguish zero from Oh, special-purpose fonts in the computer era have sometimes emphasized the difference, as recommended by proposals made in Bemer (1967). In OCR-A, a font devised for optical character recognition (OCR) by ANSI, the zero is nearly rectangular and the Oh is lozenge-shaped. The first version of the font was produced in 1968 by American Typefounders. Technological progress has made OCR-A obsolete as an OCR font, but it is still used as a display font to give a retro-techno look to images and documents. In the font OCR-B designed by Adrian Frutiger in 1968, the zero is taller and more rectangular than the oval Oh. The one, capital I, and ell are differentiated by the presence, location and angularity or curvilinearity of serifs (Figure 21). It is notable that some features of monospaced fonts designed four decades later imitate the design solutions that Frutiger devised in the 1960s (see Figure 9).

#### 5 Conclusion

Latin letters and Hindu-Arabic numerals evolved independently in handwriting in separate cultures, but each of the resulting sets of characters developed a small, open, circular or elliptical form: zero in the numerals; oh in the letters. There was no problem of confusion between the numerals and letters until they were used together in European texts in the 12th and 13th centuries, when Hindu-Arabic numerals began to be adopted by European mathematicians.

## 1I1BOOD5SZ2

## LI1BOOD5SZ2 1ILBOOD5SZ2

**Figure 21**: Traditional Typewriter revival by Monotype, OCR-A, and OCR-B.

For a few centuries, however, different contexts and, sometimes, different writing styles, appear to have been sufficient to disambiguate the meaning of the similar-looking glyphs.

In the 15th century, however, printing vastly increased the production and distribution of books and also increased pressure for standardization of character shapes when marketing books to much larger international readership. Early in the appearance of the numeral zero in print, and again near the end of the 15th century, zero took on a circular shape that was more or less unshaded, that is, without thick-thin contrast. This ring-like shape helped distinguish zero from the (typically) shaded letter oh of typefaces derived from humanist handwriting. The ring-like zero was eventually adopted for most roman typefaces in the 16th and was used until the end of the 18th and beginning of the 19th century.

In the last decades of the 18th century, a new style of numeral appeared that was more or less the same height as the capital letters. The increased height made zero easily distinguishable from oh, but created a new confusable pair: zero and capital Oh. In printing types, the zero glyph was generally narrower than Oh, so the two characters could still be distinguished, but in the last decades of the 19th century, typewriters forced all characters to have the same width, thus eliminating this width difference between zero and Oh. Context was apparently still sufficient to distinguish typewritten zero and Oh in most correspondence and documents, but in the mid-20th century, computer printers using typewriter-like fonts, and computer code that used greater mixing of letters and numerals, exacerbated the confusion. There ensued several decades of design proposals, arguments, and experiments in developing better differentiation of zero and Oh. Proposals from the "humanist" camp usually were to modify the zero, whereas proposals from the "engineer" camp were usually to modify the Oh. Technological progress in computing sometimes aided differentiation but sometimes

hindered it, whether through changes in imaging or through proliferation of confusable character forms.

In actual fonts developed for digital systems over the past three decades, signs of consensus have emerged: the zero glyph is usually the one modified, whether with a slash or a dot or other means, while the capital letter Oh is unscathed. To this extent, the humanists have been victorious. Other confusable characters have arisen, however, including the triplet of capital 'I', lowercase ell, and numeral one. Some trends have emerged among those glyphs as well, though not as clearly as for zero and Oh. Incorporation of mathematical symbols in fonts, and increasing use of mathematical symbols in electronic documents, enable further kinds of confusion, so for type designers, document designers, and readers, the problem of look-alike symbols has not been entirely solved.

#### 6 Acknowledgements

I thank Karl Berry and Barbara Beeton for suggesting this article and patiently waiting for it, and for making valuable suggestions to improve the text and images, and Karl again for putting it all into TEX format. Thanks to Steven Galbraith and Amelia Hugil-Fontanel for making the splendid resources of the RIT Cary Collection available for study of this subject, and Amelia again for photographing the page of the Hypnerotomachia Poliphili. Thanks to Rolf Rehe for help obtaining and understanding parts of DIN 1450, and to Otmar Hoefer for comments on Karlgeorg Hoefer's design of the alphabet for German vehicle license. Thanks to Kris Holmes for assembling the image for Figure 21 and for many years of inspired collaboration on the designs of the Lucida fonts.

#### 7 References

American Standards Association. American Standard Code for Information Interchange, ASA X3.4-1963, June 17, 1963.

Aristotle. Metaphysics, Books I–IX. (H. Tredennick, trans.) Loeb Classical Library No. 271, Met. 1.985b, Harvard University Press, 1989.

Bemer, R.W. Toward standards for handwritten zero and oh: Much ado about nothing (and a letter), or a partial dossier on distinguishing between handwritten zero and oh. Communications of the ACM 10(8), August 1967.

Bigelow, C. and Holmes, K. Notes on Apple 4 Fonts. Electronic Publishing 4(3), pp. 171-181, 1991. http://cajun.cs.nott.ac.uk/compsci/epo/ papers/volume4/issue3/ep050cb.pdf

Bringhurst, R. The Elements of Typographic Style. Hartley & Marks, Vancouver, CA, 1996.

Cajori, F.A. History of Mathematical Notations. Dover Publications, Mineola, NY, 1993. (Original edition: Open Court, Chicago, 1928–29.)

Deutsches Institut fur Normung. DIN 1450 Schriften— Leserlichkeit (DIN 1450 Typefaces—Legibility), DIN Beuth. 2013.

Gorn, S., Bemer, R.W., Green, J. The American Standard Code for Information Interchange. Communications of the ACM 6(8), August 1963.

Ifrah, G. The Universal History of Numbers.
(D. Bellos, E.F. Harding, S. Wood, I. Monk, trans.)
John Wiley & Sons, New York, 1998.

Kerpelman, C. Proposed American National Standard: Presentation of alphameric characters for information processing. Communications of the ACM 12(12), December 1969.

Lo, P.P. Use Chinese for zero and oh? Communications of the ACM, 10(12), December 1967.

Proposed revised American Standard Code for Information Interchange. Communications of the ACM 8(4), April 1965. Schulz-Anker, E. Syntax-Antiqua, a sans-serif on a new basis. Gebrauchsgraphik 7, 1970.

Unicode Consortium. Unicode Standard 5.0. Addison Wesley, 2007.

United States of America Standards Institute.
USA Standard Code for Information Interchange,
USAS X3.4-1967, revision of X3.4-1965, 1967.

Vartabedian, A. A Proposed Fontstyle for the Graphic Representation of the Oh and Zero. The Journal of Typographic Research 3(3), pp. 249–258, 1969.

Vartabedian, A. Reply. The Journal of Typographic Research 4(2), pp. 181–183, 1970.

Wendt, D. O or 0. The Journal of Typographic Research 3(3), pp. 241–248, 1969.

Zapf, H. Proposal. The Journal of Typographic Research 4(2), pp. 179–180, 1970.

#### Production notes

Karl Berry

The most TEXnically unusual part of this article, and of the entire issue, was handling the rare characters shown in the footnote on the first page and the rundown of circle-slash characters on the next two pages. Although they could have been inserted as small images, the author (Chuck Bigelow) sent me fonts including them, so I wanted to try typesetting them directly. He wanted to typeset them all in a consistent font, rather than mixing glyphs from Computer Modern and other sources.

The first version Chuck sent me was in .otf format, with the characters we wanted (zero-slash, prohibition, etc.) replacing lowercase letters. So it sufficed to start up FontForge (by George Williams, fontforge.sf.net) and use its 'Generate Fonts' feature to create a .pfb+.afm, which takes the first 256 characters. Easy. (I wanted to use Type 1 since this was happening quite far along in the article's processing, and I had been using pdflATeX thus far; switching to XqLATeX or LualATeX would have meant losing functionality from microtype and thus losing considerable time fixing line breaks.)

Then Chuck sent me a revised font with additional characters. This time it was a .ttf, and the characters were in the correct Unicode positions (which are far beyond the first 256 characters, of course), so I couldn't just use the simple FontForge generation. (I could have asked Chuck to rearrange the characters, but I decided to take it as a challenge; after all, it's an article of our faith that TEX should be able to use any font.)

Instead, I followed the article by Hàn Thế Thành about using TrueType fonts directly in pdfTEX (30:1, tug.org/TUGboat/tb30-1/tb94thanh.pdf). First I created a custom encoding file, altzero.enc, starting like this:

These character names are specified in the font. I discovered them by looking at the font in FontForge and using

'View  $\rightarrow$  Goto' to navigate to the characters; thankfully, searching for uni... works even when the character does not have a name of that form. Chuck told me the name of the variant emptyset glyph (zero-slash in this case), which does not have a Unicode assignment.

Still following Thành's article, I then made the .tfm:

 $\label{tf2afm} \mbox{-e altzero.enc -o altzero.afm ZeroFont.ttf} $$ afm2tfm altzero.afm $$ $$ 

In the LATEX document, the font was used like this:

\font\altzero = altzero {\altzero\char0}% of our encoding: emptyset

All was fine, until Chuck sent me one more revision of the font. This time it was again .otf, but now using the Unicode positions. pdfTEX cannot read .otf, and converting .otf to .ttf seemed fraught with potential problems to me. So I used a third tool: otftotfm (by Eddie Kohler, lcdf.org). Once I read the documentation enough times, I happily discovered that I could re-use the same encoding file. The invocation this time:

```
otftotfm -e altzero.enc --no-encoding \
ZeroFontOT.otf altzero
```

(The --no-encoding option just tells offtoffm not to generate its own new encoding file; the final altzero argument is the base name of the .tfm and .pfb generated by offtoffm.)

Usage in the LATEX source is similar to the above, but now we have a .pfb:

The tools themselves output the map lines needed, according to the names embedded in the font files, etc.

Moving on from the technicalities, it was a great pleasure to work with Chuck on his articles in this issue. He has had a great (and positive!) influence on me, with recommendations for schools to attend, professors to work with, and personally encouraging my lifelong interest in typography and typesetting. As it turned out, we effectively finished work on the article on Chuck's birthday. Happy birthday Chuck!