plots. They are always used with the globally defined options

2 The parallel projection

Figure 1 shows a point P(x,y,z) in a three dimensional cartesian coordinate system (x,y,z) with a transformation into $P^*(x^*,y^*)$, the point in the two dimensional system (x_E,y_E) .

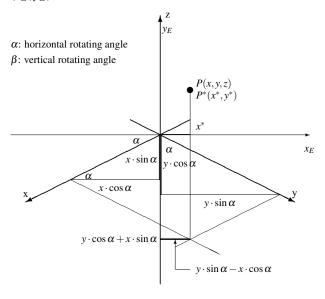


Figure 1: Lengths in a three dimensional system

The angle α is the horizontal rotation with positive values for anti-clockwise rotations of the 3D coordinates. The angle β is the vertical rotation (orthogonal to the paper plane). In figure 2 we have $\alpha = \beta = 0$. The y-axis comes perpendicularly out of the paper plane. Figure 3 shows the same for another angle with a view from the side, where the x-axis shows into the paper plane and the angle β is greater than 0 degrees.

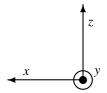


Figure 2: Coordinate system for $\alpha = \beta = 0$ (y-axis comes out of the paper plane)

The two dimensional x coordinate x^* is the difference of the two horizontal lengths $y \cdot \sin \alpha$ and $x \cdot \cos \alpha$ (figure 1):

$$x^* = -x \cdot \cos \alpha + y \cdot \sin \alpha \tag{1}$$

The z-coordinate is unimportant, because the rotation comes out of the paper plane, so we have only a

Three dimensional plots with pst-3dplot

Herbert Voß

Abstract

The well-known pstricks package [7] offers excellent macros for creating more or less complex graphics which could be inserted into the document without having it exported to EPS or PDF. pstricks itself is the base for several other additional packages, which are typically named pst-xxxx, such as pst-3dplot.

There exist several packages for plotting three dimensional graphical objects. pst-3dplot handles three dimensional objects, mathematical functions, and data files similarly to pst-plot in two dimensions.

1 Introduction

The pstricks packages are available as usual from any possible CTAN server. The base parts are located at CTAN:graphics/pstricks/generic/ and most of the additional packages at CTAN:graphics/pstricks/contrib/[7].

All \psgrid commands are only for a better view of the examples, they are not really necessary for the 3D-

different y^* value for the two dimensional coordinate but no other x^* value. The β angle is well seen in figure 3 which derives from figure 2, if the coordinate system is rotated by 90 deg horizontally to the left and vertically by β also to the left.

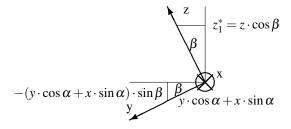


Figure 3: Coordinate system for $\alpha = 0$ and $\beta > 0$ (x-axis goes into the paper plane)

The value of the perpendicular projected z coordinate is $z^* = z \cdot \cos \beta$. With figure 3 we see that the point P(x,y,z) runs on an elliptical curve when β is constant and α changes continously. The vertical alteration of P is the difference of the two "perpendicular" lines $y \cdot \cos \alpha$ and $x \cdot \sin \alpha$. These lines are rotated by the angle β , so we have to multiply them with $\sin \beta$ to get the vertical part. We get the following transformation equations:

$$x_E = -x\cos\alpha + y\sin\alpha$$

$$y_E = -(x\sin\alpha + y\cos\alpha) \cdot \sin\beta + z\cos\beta$$
 (2)

or the same written in matrix form:

$$\begin{pmatrix} x_E \\ y_E \end{pmatrix} = \begin{pmatrix} -\cos\alpha & \sin\alpha & 0 \\ -\sin\alpha\sin\beta & -\cos\alpha\sin\beta & \cos\beta \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$
(3)

3 Coordinate axes

The syntax for drawing the coordinate axes is

\pstThreeDCoor[<options>]

Without any options, we get the default view seen in figure 4 with the predefined values:

```
xMin=-1, xMax=4,
yMin=-1, yMax=4,
zMin=-1, zMax=4,
Alpha=45, Beta=30
```

There are no restrictions for the angles and the max and min values for the axes; all pstricks options are possible as well. The following example (5) changes the color and the width of the axes. The angles Alpha and Beta are important to all macros and should always be set with psset to make them global to all other macros. Otherwise they are only local inside the macro to which they are passed.

```
begin{pspicture}(-2,-1)(1,2.25)
psgrid
```

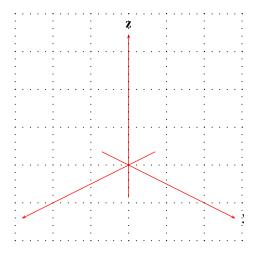


Figure 4: The default 3D coordinate system

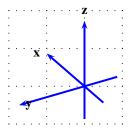


Figure 5: Axes with a different view and color

4 put command

The syntax is similar to the \rput macro from the package pst-plot:

```
\pstThreeDPut[<options>]%
  (x,y,z){<any material>}
```

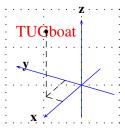


Figure 6: Example for the \pstThreeDPut macro

```
l \begin{pspicture} (-2,-1)(1,2.25)
```

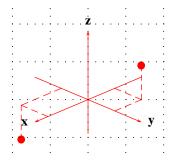


Figure 7: 3D dots with marked coordinates

```
psgrid
psset{ Alpha=-60, Beta=-30}

pstThreeDCoor[%

inecolor=blue, %

xMin=-1, xMax=2,

yMin=-1, yMax=2, %

zMin=-1, zMax=2]

pstThreeDPut(1,0.5,2) {\red\large TUGboat}

pstThreeDDot[drawCoor=true](1,0.5,2)

end(pspicture)
```

Internally, the \pstThreeDPut macro defines a two dimensional node temp@pstNode and then uses the default \rput macro from pstricks. Because of the perspective from which the coordinate system is viewed, the 3D dot will not be seen as the center of the printed material when this is also a three dimensional one. This does not happen for figure 6, because the text is only a two dimensional object.

5 Nodes

The syntax is

```
\pstThreeDNode(x,y,z) {<node name>}
```

This node is internally transformed into a two dimensional node, so it cannot be used as a replacement for the parameters (x,y,z) of the 3D dot which is possible with the macros from pst-plot. If A and B are two nodes, then \psline{A}{B} draws a line from A to B. Doing the same with pst-3dplot is not yet implemented. On the other hand, it is not a problem to define two 3D nodes C and D and then draw a two dimensional line from C to D.

6 Dots

The syntax for a dot is

```
\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\protect\pro
```

Dots can be drawn with dashed lines for the three coordinates, when the option drawCoor is set to true (figure 7).

In the figure 8 the coordinates of the dots are (a, a, a) where a is -3, -2, -1, 0, 1, 2, 3.

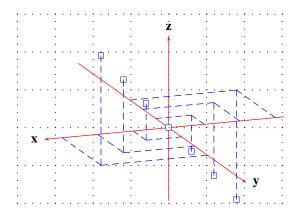


Figure 8: Another demonstration for drawing dots

7 Lines

The syntax for a three dimensional line is

```
\pstThreeDLine[<options>]% (x1,y1,z1)(x2,y2,z2)
```

All options for lines from pst-plot are possible, there are no special ones for a 3D line. The only difference in drawing a line or a vector is that the first one has an arrow of type – and the second type –> (figure 9).

```
\psset{xMin=-2, xMax=2, yMin=-2, yMax=2, %
            zMin=-2.zMax=2
    \begin{pspicture} (-2, -2.25) (2, 2.25)
       \pstThreeDCoor
       \psset{dotstyle=*,linecolor=red, %
              drawCoor=true}
       \protect\
       \protect\operatorname{\mathtt{NPStThreeDDot}}(1.5,-1,-1)
       \pstThreeDLine[%
10
           linewidth=3pt,%
11
           linecolor=blue,
12
           arrows=->%
        ](-1,1,0.5)(1.5,-1,-1)
```

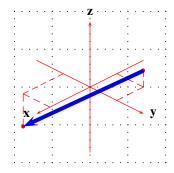


Figure 9: Drawing a 3D vector

```
14 \psgrid
15 \end{pspicture}
```

8 Triangle

A triangle is given by its three points:

```
\pstThreeDTriangle[<options>](P1)(P2)(P3)
```

When the option fillstyle is set to value other than none, the triangle is filled with the active color or with the one which is set with the option fillcolor (figure 10).

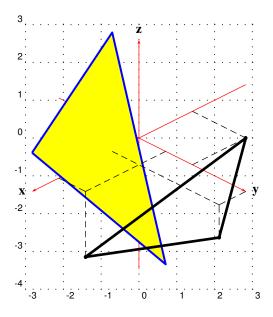


Figure 10: Triangles with fill option

For triangles especially, the option linejoin is important. Its value is passed to the PostScript command setlinejoin. The default value is 1, which gives rounded edges (figure 11).

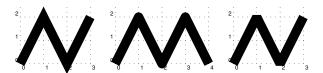


Figure 11: Meaning of the PostScript command setlinejoin=0|1|2

9 Squares

The syntax for a 3D square is:

```
\pstThreeDSquare%
  [<options>]
  (<vector o>)%
  (<vector u>)(<vector v>)
```

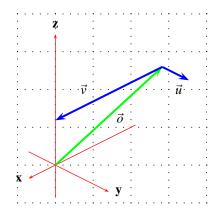


Figure 12: Drawing a square with three vectors

Squares are nothing more than a polygon with the starting point P_o given with the origin vector \vec{o} and the two direction vectors \vec{u} and \vec{v} , which build the sides of the square as shown in figure 12. With the fillstyle option the square can be filled with the in pst-plot defined styles, for example solid like in figure 13. All the options of pstricks are allowed for this macro.

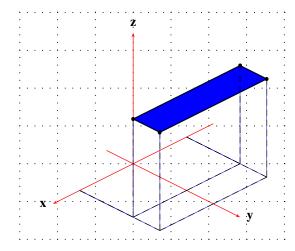


Figure 13: Drawing a filled square with the vectors from figure 12

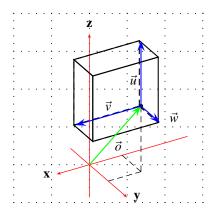


Figure 14: Drawing a box with three vectors

10 Boxes

A box is a special case of a square and has the syntax

```
\pstThreeDBox%
  [<options>]
  (<vector o>%
  (<vector u>) (<vector v>) (<vector w>)
```

All options from pstricks are possible here. The other parameters are the origin vector \vec{o} and the three direction vectors \vec{u} , \vec{v} and \vec{w} . The figure 14 shows a box together with these four vectors. In this example the three direction vectors are perpendicular to each other.

```
9  \uput[0](0.9,2.25){$\vec{u}$}
10  \uput[90](0.5,1.25){$\vec{v}$}
11  \uput[45](2,1.){$\vec{w}$}
12  \pstThreeDLine[linecolor=green](0,0,0)(-1,1,2)
13  \pstThreeDLine[linecolor=blue](-1,1,2)(-1,1,4)
14  \upstThreeDLine[linecolor=blue](-1,1,2)(1,1,2)
15  \upstThreeDLine[linecolor=blue](-1,1,2)(-1,2,2)
16  \end{pspicture}
```

11 Ellipses and circles

The equation for a two dimensional ellipse (figure 15) is:

$$e: \frac{(x-x_M)^2}{a^2} + \frac{(y-y_M)^2}{b^2} = 1$$
 (4)

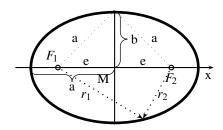


Figure 15: Definition of an ellipse

 $(x_m; y_m)$ is the center, a and b the eccentricity. For a = b = 1 in equation 4 we get the "one" for the circle, which is nothing more than a special case of an ellipse. The equation written in parametric form is

$$x = a \cdot \cos \alpha$$

$$y = b \cdot \sin \alpha$$
 (5)

or the same with vectors to get an ellipse in a 3D system:

$$e: \vec{x} = \vec{c} + \cos \alpha \cdot \vec{u} + \sin \alpha \cdot \vec{v}$$

$$0 < \alpha < 360 \quad (6)$$

where \vec{c} is the center, \vec{u} and \vec{v} the directions vectors which must be perpendicular to each other.

11.1 Options

In addition to all possible options from the package pst-plot, we have two special ones for the drawing of an arc (with predefined values for a full ellipse or circle):

```
beginAngle=0
endAngle=360
```

Using the parametricplotThreeD macro (described in section 13.2, ellipses and circles are drawn with a default setting of 50 points for the ellipse or circle.

11.2 Ellipse

In a 3D coordinate system, it is very difficult to see the difference between an ellipse and a circle. Depending on the point of view an ellipse may be seen as a circle and vice versa (figure 16). The syntax of the ellipse macro is:

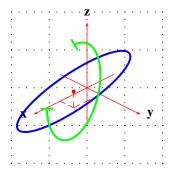


Figure 16: Drawing ellipses

```
\pstThreeDEllipse%
  [<options>]%
  (cx,cy,cz)%
  (ux,uy,uz)(vx,vy,vz)
```

where c is for center and u and v for the two direction vectors (eq. 6).

```
\psset{xMin=-1,xMax=2,yMin=-1,yMax=2,zMin=-1,zMax}
        =2}
   \begin{pspicture} (-2,-2)(2,2)
       \psgrid
      \pstThreeDCoor
      \pstThreeDDot[%
          linecolor=red.%
          drawCoor=true](1,0.5,0.5)% the center
      \pstThreeDEllipse[%
          linecolor=blue, linewidth=1.5pt]%
          (1,0.5,0.5)(-0.5,1,0.5)(1,-0.5,-1)
10
11
      % settings for an arc
      \pstThreeDEllipse[%
           beginAngle=0,endAngle=270,%
13
           linecolor=green]%
14
           (1,0.5,0.5)(-0.5,0.5,0.5)(0.5,0.5,-1)
15
   \end{pspicture}
```

11.3 Circle

The circle is a special case of an ellipse (eq. 6) with the vectors \vec{u} and \vec{v} which are perpendicular to each other: $|\vec{u}| = |\vec{v}| = r$. with $\vec{u} \cdot \vec{v} = \vec{0}$

The macro \pstThreeDCircle is nothing more than a synonym for \pstThreeDEllipse. In the following example the circle is drawn with only 20 plotpoints and the option showpoints=true.

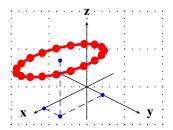


Figure 17: Drawing a circle with the option showpoints

12 Spheres

Internally, pst-3dplot uses the macro from the pst-vue3d package¹ to draw spheres, and places it with the \rput macro at the right place. The syntax for this macro is

```
\pstThreeDSphere[<options>](x,y,z){Radius}
```

(x, y, z) is the center of the sphere. For all the other possible options or the possibility to draw demispheres, refer to the documentation.[3]

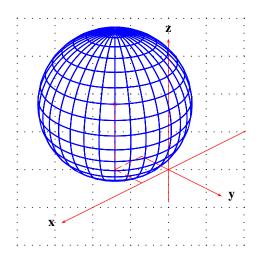


Figure 18: Drawing a sphere with package pst-vue3d

 $^{^{\}rm 1}$ CTAN:graphics/pstricks/contrib/pst-vue3d, and from Manuel Luque's homepage[3]. The documentation is in French, but it is mostly self-explanatory.

13 Mathematical functions

There exist two macros for plotting mathematical functions f(x,y), which work similarly to the one from pst-plot.

13.1 Function f(x,y)

The macro for plotting functions does not have the same syntax as the one from pst-plot[5], but it is used in the same way:

```
\psplotThreeD[<options>]%
    (xMin,xMax)(yMin,yMax)%
    {<the function>}
```

The function has to be written in PostScript code and the only valid variable names are x and y. For example, $\{x \text{ dup mul } y \text{ dup mul add sqrt} \}$ represents the math expression $\sqrt{x^2+y^2}$. The macro \psplotThreeD has the same plotstyle options as \psplot, except the plotpoints-option which is split into one for x and one for y (table 1).

Table 1: Options for the plot macros

Option name	value
plotstyle	dots
	line
	polygon
	curve
	ecurve
	ccurve
	none (default)
showpoints	default is false
xPlotpoints	default is 25
yPlotpoints	default is 25
hiddenLine	default is false

Equation 7 is plotted with the following parameters and seen in figure 19.

$$z = 10\left(x^3 + xy^4 - \frac{x}{5}\right)e^{-\left(x^2 + y^2\right)} + e^{-\left((x - 1.225)^2 + y^2\right)}$$

```
\begin{pspicture} (-6,-4) (6,5)
        \psarid
2
        \psset{Alpha=45,Beta=15}
        \psplotThreeD[%
            plotstyle=line, %
             yPlotpoints=40,xPlotpoints=30,%
             linewidth=1pt](-4,4)(-4,4){%
                x 3 exp x y 4 exp mul add x 5 div sub
                        10 mul
                 2.729 \ \text{x} \ \text{dup} \ \text{mul} \ \text{y} \ \text{dup} \ \text{mul} \ \text{add} \ \text{neg} \ \text{exp}
                       mul
                 2.729 \times 1.225 sub dup mul y dup mul add
10
                      neg exp add}
11
        \pstThreeDCoor[xMin=-1,xMax=5,yMin=-1,yMax=5,
              zMin=-1, zMax=5]
    \end{pspicture}
```

The function is calculated within two loops:

```
for (float y=yMin; y<yMax; y+=dy)
  for (float x=xMin; x<xMax; x+=dx)
    z=f(x,y);</pre>
```

Because of the inner loop it is only possible to get a closed curve in x direction. Therefore fewer yPlotpoints are not a real problem, but too few xPlotpoints results in a bad drawing of the mathematical function, especially for the plotstyle option line.

Drawing three dimensional mathematical functions with curves which are transparent makes it difficult to see if a point is before or behind another one. \psplotThreeD has an option hiddenLine for a primitive hidden line mode, which only works well when the y-interval is defined such that $y_2 > y_1$. Then, every new curve is plotted over the previous one and filled with the color white. Figure 20 is the same as figure 19, only with the option hiddenLine=true.

13.2 Parametric plots

Parametric plots are possible for drawing curves or areas. The syntax for this plot macro is:

```
\parametricplotThreeD[<options>]%
  (t1,t2)(u1,u2)%
  {<three parametric functions x y z}</pre>
```

The only possible variables are t and u with t1, t2 and u1, u2 as the range for the parameters. The order for the functions is not important and u may be optional when having only a three dimensional curve and not an area.

$$x = f(t,u)$$

$$y = f(t,u)$$

$$z = f(t,u)$$
(8)

To draw a spiral we have the parametric functions:

$$x = r\cos t$$

$$y = r\sin t$$

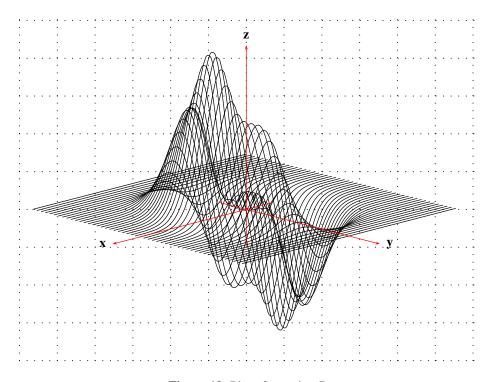
$$z = t/600$$
(9)

In the example, the t value is divided by 600 for the z coordinate, because we have the values for t in degrees, here with a range of $0^{\circ} \dots 2160^{\circ}$. Drawing a curve in a three dimensional coordinate system does only require one parameter, which is by default t. In this case we do not need all parameters, so that we can write

```
\parametricplotThreeD[<options>]%
  (t1,t2)%
  {<three parametric functions x y z}</pre>
```

which is the same as (0,0) for the parameter u. Figure 21 shows a three dimensional curve.

```
1 \begin{pspicture}(-3,-2)(3,5)
2 \psgrid
3 \parametricplotThreeD[%
4 xPlotpoints=200,%
5 linecolor=blue,%
```



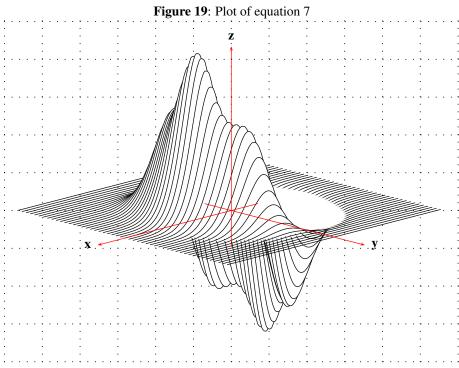


Figure 20: Plot of equation 7 with the hiddenLine=true option

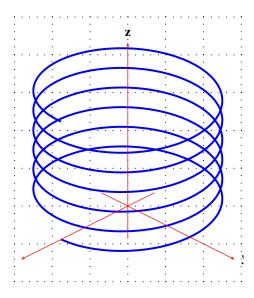


Figure 21: Drawing a 3D curve

Instead of using the \pstThreeDSphere macro (see section 12) it is also possible to use parametric functions for a sphere. The macro plots continous lines only for the t parameter, so a sphere plotted with the longitudes needs the parametric equations as

$$x = \cos t \cdot \sin u$$

$$y = \cos t \cdot \cos u$$

$$z = \sin t$$
(10)

The same is possible for a sphere drawn with the latitudes:

$$x = \cos u \cdot \sin t$$

$$y = \cos u \cdot \cos t$$

$$z = \sin u$$
(11)

and lastly, we can have both of these parametric functions together in one pspicture environment (figure 22).

```
\begin{pspicture}(-1,-1)(1,1)
     \psgrid
   \parametricplotThreeD[ %
      plotstyle=curve, yPlotpoints=40](0,360)(0,360){%
         t cos u sin mul
6
         t cos u cos mul
         t sin
9
   \parametricplotThreeD[%
10
      plotstyle=curve, yPlotpoints=40](0,360)(0,360){%
11
        u cos t sin mul
         u cos t cos mul
12
         u sin
```



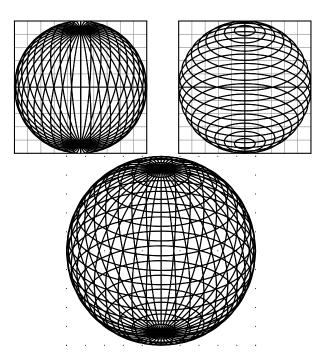


Figure 22: Different views of the same parametric functions

14 Plotting data files

We have the same conventions for data files which hold 3D coordinates as for 2D. For example:

```
0.0000 1.0000 0.0000

-0.4207 0.9972 0.0191

....

0.0000, 1.0000, 0.0000

-0.4207, 0.9972, 0.0191

....

(0.0000,1.0000,0.0000)

(-0.4207,0.9972,0.0191)

....

{0.0000,1.0000,0.0000}

{-0.4207,0.9972,0.0191}

....
```

There are the same three plot functions:

```
\fileplotThreeD[<options>]{<datafile>}
\dataplotThreeD[<options>]{<data object>}
\listplotThreeD[<options>]{<data object>}
```

The data file used in the following examples has 446 entries like

```
6.26093349..., 2.55876582..., 8.131984...
```

Using the listplotThreeD macro with many data entries may take considerable time on slow machines. The possible options for the lines are the same as earlier, given in table 1.

14.1 \fileplotThreeD

The syntax is straightforward:

```
\fileplotThreeD[<options>] {<datafile>}
```

If the data file is not in the same directory as the document, use the file name with the full path. Figure 23 shows a file plot with the option linestyle=line.

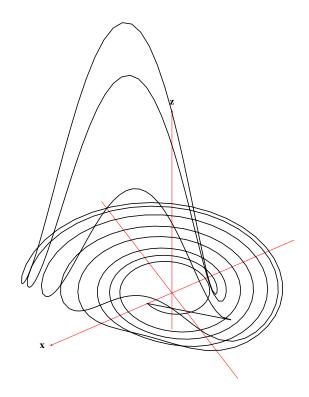


Figure 23: Demonstration of \fileplotThreeD with Alpha=30 and Beta=15

14.2 \dataplotThreeD

The syntax is:

\dataplotThreeD[<options>]{<data object>}

In contrast to \fileplot ThreeD, the second macro \dataplot ThreeD reads the data entries from another

macro. Using $\$ readdata, external data can be read from a file and saved in a macro, to be passed to $\$ dataThreeD[1].

\readdata{<data object>}{<datafile>}

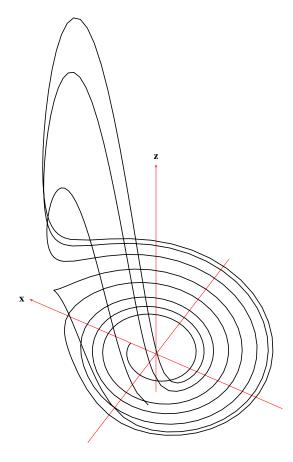


Figure 24: Demonstration of \dataplotThreeD with Alpha=-30 and Beta=30

14.3 \listplotThreeD

The syntax is:

```
\listplotThreeD[<options>]{<data object>}
```

There is no essential difference between the macros \istplotThreeD and \dataplotThreeD. With \listplotThreeD, one can pass additional PostScript code, which is appended to the data object. For example:

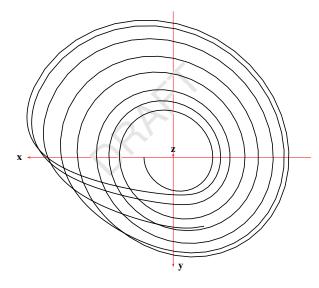


Figure 25: Demonstration of \listplotThreeD with a view from above (Alpha=0 and Beta=90) and some additional PostScript code

Figure 25 shows what happens with this additional PostScript code. Another example can be found in [5], where ScalePoints is redefined. For pst-3dplot, the equivalent macro is named ScalePointsThreeD.

15 PDF output

pst-3dplot is based on the popular pstricks package and writes pure PostScript code[2], so it is not possible to run TEX files with pdfLATEX when there are pstricks macros in the document. If you need PDF output, there are the following possibilities:

- the package pdftricks.sty [6]
- the free (for Linux only) program VTEX/Lnx (http://www.micropress-inc.com/linux/
- the ps2pdf (dvi→ps→pdf) or dvipdfm utilities
- the ps4pdf package [4].

If you need package graphicx.sty, load it before any pstricks package. You do not need to load pstricks.sty, as this will be done by pst-3dplot.

References

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- [5] Herbert Voß. Die mathematischen Funktionen von PostScript. *Die TEXnische Komödie*, 1/02:40–47, March 2002.
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