Math into BLUes

Part I: Mourning

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Abstract

TEXing mathscripts is not simply typing. Math has to be translated into TEX commands. First the motivation for this work is given. Next traditional math page makeup is summarized along with the macroscopic math TEX commands. After answering "Why is TEXing mathscripts difficult?", an anthology of TEXfalls and their antidotes is discussed. In part II, suggestions are given in order to lessen the difficulties.

Prelude

My assistance was called for in TEXing a mathscript. Part of the mathscript was typed, and contained TEX commands; but it did not compile. Inspection revealed it never could have. It occurred to me that at least three typists had been involved, mixing the use of LATEX, AMS-TEX and macros from other sources. Furthermore, the TEXscript showed various 'TEXfalls' (from "pitfalls"). I define these as "correct encoding which yields neither the required nor customary layout." Also 'pseudo-guru' involvement could be felt, which I would define as a too-complicated use of TEX, inhibiting the intelligibility of the TEXscript. There is an appropriate quote to be found on page 373 of *The TEXbook* in the "Dirty Tricks" chapter.

> Always remember, however, that there's usually a simpler and better way to do something than the first way that pops into your head.

Not only was I looking over the shoulder of a typist, I was also inspecting a math book TEXed by a mathematician (Temme, 1990). The book looks good and examples from it are used here in order to show other ways of TEXing.

In Part I, attention is paid to:

- traditional math page make-up;
- some advanced math examples;
- what makes TEXing mathscripts difficult; and
- an anthology of TEXfalls with antidotes.

Part II of this series will deal with:

- (cross)referencing;
- hyphenation of long formulae; and
- what ought to be done to lessen the difficulties.

Part II will be published in the Proceedings of EuroTEX91, Paris, Cahiers GUTenberg, #10–11, 147–170.

With respect to the future TEXing of math I don't consider Mittelbach's (1990:11) criticisms too severe. First, the spacing around atoms can be adapted via casts. Second, the lack of hyphenation for subformulae is similar to verbatim; in general, hyphenation is avoided in boxes. Math hyphenation has been conscientiously avoided in displays as well. For in-line math, it is true that *sub*formulae are not hyphenated automatically. It is not that relevant however, because in-line math should be short, and should not be complicated (read 'nested').

For you and me. Most, if not all, math TEXfalls[1] have been envisioned by the Grand Wizard himself and references to those or related issues are indicated by '*TB*' (*The TEXbook*) followed by page or exercise number. Other terms used in this paper include the following. 'Mathscript' denotes a mathematics manuscript. 'TEXscript' denotes a TEX formatted compuscript, especially the one for which my assistance was asked. 'TEXnigma' is a computer system with TEX installed. 'TEXnowledge' means knowledge of TEX. A 'TEXist' is a TEX typist. 'DEK' is Donald E. Knuth. 'BLUe' is DEK's

[1] The TEXfalls discussed herein are not specific to plain TEX, AMS-TEX, or LATEX. They illustrate basic pitfalls in encoding math. Sources include the inspected TEXscript, the Temme book and some pitfalls I stumbled upon myself. LATEX is rather superficial with respect to math. Formula classes are not even mentioned, which is dead wrong, but understandable from the viewpoint of descriptive markup.

unwary B.L. User (Ben Lee User of *The TEXbook* fame). 'TEXfalls' has already been described.

Math Page Makeup

Swanson (1986) is a good source for information on traditional math markup. In publications, math is either part of the running text or is displayed. In displays, indentation on all sides is active, and formulae are sometimes aligned, for example, at the = symbol.

TEX requires math within text to be surrounded by $ signs:[2] $<math>$. Displayed math is tagged by $$ signs:[3] $$<displayed math>$$. For the general multi-line display, plain TEX provides the macro `\displaylines` (*TB* 194, 362), and for aligned formulae the macro `\eqalign` (*TB* 190, 362). Displays are centered by default and that is all there is to TEXing math, from an outer level point of view.

- The following example of a Pascal triangle:

$$\displaylines{1\cr 1\quad1\cr 1\quad2\quad1\cr 1\quad3\quad3\quad1\cr \cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot}$$

is obtained via:

```
$$\displaylines{1\cr
  1\quad1\cr
  1\quad2\quad1\cr
  1\quad3\quad3\quad1\cr
  \hbox to 7em{$\cdot$\hss
    $\cdot$\hss$\cdot$\hss
        $\cdot$\hss$\cdot$}}$$
```

The example demonstrates two levels of formatting math: (1) the inner level, where the triangle has to be defined unambiguously (detailed TEX commands), and (2) the outer level, where the triangle is positioned within the text ($$ signs meaning display) and subject to the style of the publication series.[4]

On the phone one would say: "Pascal's triangle; you know; a 1 with two 1's below it, and a 1,2,1 below that, and a 1,3,3,1 below that, etc., all centered." However, for formatting (read 'encoding'), more precise information is needed than when describing math by phone, in order to eliminate ambiguity. A computer-based formatting system is not yet that intelligent.

Right- or left-aligned formula numbers can be provided by the tags `\eqno` and `\leqno` (*TB* 187, 362). Individual lines in a multi-line display can be numbered; therefore, the macros `\eqalignno` and `\leqalignno` are provided (*TB* 192, 362).

- In summary, all plain TEX's math page makeup macros (with essential ways of numbering formulae) are demonstrated in the following templates:

$$\sin 2x = 2\sin x\, \cos x \qquad \text{(TB 186)}$$

$$F(z)=a_0+{a_1\over z}+{a_2\over z^2}+\cdots+{a_{n-1}\over z^{n-1}}+R_n(z), \qquad n=0,1,2,\dots\,,$$

$$F(z)\sim\sum_{n=0}^\infty a_nz^{-n}, \quad z\to\infty \qquad \text{(TB ex19.16)}$$

$$\begin{aligned}\cos 2x&=2\cos^2x-1\\ &=\cos^2x-\sin^2x\end{aligned} \qquad \text{(TB 193)}$$

$$\begin{aligned}\cosh 2x&=2\cosh^2x-1 && \text{(TB 192)}\\ &=\cosh^2x+\sinh^2x\end{aligned}$$

which are obtained via:

```
$$\sin2x=2\sin x\, \cos x
     \eqno({\rm TB\ 186})$$
$$\displaylines{F(z)=
a_0+{a_1\over z}+{a_2\over z^2}+\cdots
   +{a_{n-1}\over z^{n-1}}+R_n(z),\cr
           \hfill n=0,1,2,\dots\,,\cr
\hfill F(z)\sim\sum_{n=0}^\infty a_nz^{-n},
       \quad z\to\infty\qquad\qquad\hfill
       \llap{(TB\ ex19.16)}\cr}$$
$$\eqalign{\cos2x&=2\cos^2x-1\cr
                &=\cos^2x-\sin^2x\cr}
  \eqno({\rm TB\ 193})$$
$$\eqalignno{
\cosh2x&=2\cosh^2x-1&({\rm TB\ 192})\cr
       &=\cosh^2x+\sinh^2x\cr}$$
```

It was difficult to get the above example `\eqalign`, labeled TB 193, to work correctly in two-column format. It would left-justify rather than center the formula because of insufficient space left by the wide label. Deactivating the glue '`\,`' before the `\vcenter` in the body of `\eqalign` forced TEX to center the formula (see *TB* 189).

One can also use the general `\halign` macro.

- From example 22.9 of *The TEXbook*, we have:

---

[2] Expensive!

[3] Even more expensive!

[4] What should be displayed is left to the discretion of the author but it should serve clarity of exposition. Swanson (1986) advises displaying any math which is longer than half a line.

$$\begin{aligned} 10w + 3x + 3y + 18z &= 1, & (9)\\ 6w - 17x \quad - 5z &= 2, & (10)\end{aligned}$$

obtained via:

```
$$\openup1\jot\tabskip=0pt plus1fil
\halign to\displaywidth{\tabskip=0pt
$\hfil#$&$\hfil{}#{}$&
$\hfil#$&$\hfil{}#{}$&
$\hfil#$&$\hfil{}#{}$&
$\hfil#$&${}#\hfil$\tabskip=0pt plus1fil&
\llap{#}\tabskip=0pt\cr
10w&+& 3x&+&3y&+&18z&=1,&( 9)\cr
 6w&-&17x& &  &-& 5z&=2,&(10)\cr}$$
```

I consider `\cases`, `\(p)matrix`, `\overbrace`, and `\underbrace` to be *parts* of formulae (*TB* 177):

- `\(p)matrix` as formula part

$$A = \begin{pmatrix} a_{11} & a_{12} & \ldots & a_{1n}\\ a_{21} & a_{22} & \ldots & a_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ a_{n1} & a_{n2} & \ldots & a_{nn}\end{pmatrix}$$

obtained via:

```
$$A=\pmatrix{
    a_{11}&a_{12}&\ldots&a_{1n}\cr
    a_{21}&a_{22}&\ldots&a_{2n}\cr
    \vdots&\vdots&\ddots&\vdots\cr
    a_{n1}&a_{n2}&\ldots&a_{nn}\cr}$$
```

Am I BLUe?

Before diving into the difficulties which arise when encoding math, the following are some examples which show what TeX can do for you.[5]

- Selections from chapters 16–18 in *The TeXbook*:

$$\mathbf{S}^{-1}\mathbf{TS} = \mathbf{dg}(\lambda_1, \ldots, \lambda_n) = \boldsymbol{\Lambda}$$

$$\sum_{k=1}^{\infty} \frac{1}{2^k} = 1, \quad \sqrt{1+\sqrt{1+\sqrt{1+x}}}$$

$$\frac{f(x+\Delta x) - f(x)}{\Delta x} \to f'(x) \quad \text{as} \quad \Delta x \to 0$$

$$\{\underbrace{\overbrace{a,\ldots,a}^{k\ a\text{'s}},\overbrace{b,\ldots,b}^{l\ b\text{'s}}}_{k+l\ \text{elements}}\}, \quad 2\uparrow\uparrow k \stackrel{\text{def}}{=} \left.2^{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}}\right\}k$$

- The Cardano solution of $x^3+px=q$, with $p,q \geq 0$ reads:

$$\sqrt[3]{\sqrt{p^3/27 - q^2/4} + q/2} - \sqrt[3]{\sqrt{p^3/27 + q^2/4} - q/2}$$

[5] NTG (Dutch Users Group) uses this section, along with other interesting examples, in their info/demo package for potential members.

- Derivatives:[6]
  $\dot y\, \ddot y\, \dddot y \quad y'\, y''\, y''' \quad \partial_x y\, \partial_x^2 y, \partial_x^3 y$
- Bessel equation:

$$z^2w'' + zw' + (z^2 - \nu^2)w = 0$$

  with solutions:
  $J_{\pm\nu}(z)$, Bessel function of the first kind,
  $Y_{\pm\nu}(z)$, Bessel function of the second kind (Weber),
  $H_\nu^{(1)}(z)$, and $H_\nu^{(2)}(z)$, Bessel function of the third kind (Hankel).
- Primed summation symbols are used in Chebyshev expansions:

$$\sum_{k=0}^{n}{}' a_kT_k(x) \stackrel{\text{def}}{=} .5a_0 + a_1x + a_2T_2(x) + \cdots + a_nT_n(x).$$

- Hypergeometric function:

$$M_n(z) = {}_{n+1}F_n\left({k+a_0, k+a_1, \ldots, k+a_n \atop k+c_1, \ldots, k+c_n}; z\right)$$

- From Swanson (1986:40):

$$\begin{aligned}&\int_U \delta(I)\mu(I) \leq \\ &\sum_{\mathcal{D}}\sum_{\mathcal{D}_{I'}}\Bigg[\int_J \alpha(J')\mu(J') - \alpha(J)\mu(J) \\ &\qquad - \int_J [\{s(\alpha\eta)(J')\}/\eta(J')]\mu(J')\Bigg] \\ &+ \left[\sum_{\mathcal{D}}\sum_{\mathcal{D}_{I'}} |\alpha(J) - [\{s(\alpha\eta)(J)\}/\eta(J)]|\mu(J)\right] \\ &\times \left[\sum_{\mathcal{D}}\sum_{\mathcal{D}_{I'}} |\alpha(J) - [\{s(\alpha\eta)(J)\}/\eta(J)]|\eta(J)\right]\end{aligned}$$

- Magic squares (Dürer's 4-by-4 with dotted lines):

2	7	6
9	5	1
4	3	8

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

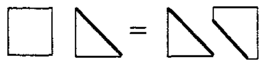
- Calculation flow of autocorrelation function a_f (inspired by *TB* 358, ex18.46). $\mathcal{F}$ denotes Fourier transform and $\mathcal{F}^-$ the inverse Fourier

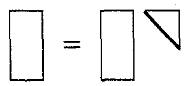
[6] Kerning an extra point in superscripts was pointed out by Daniel Olson.

transform:

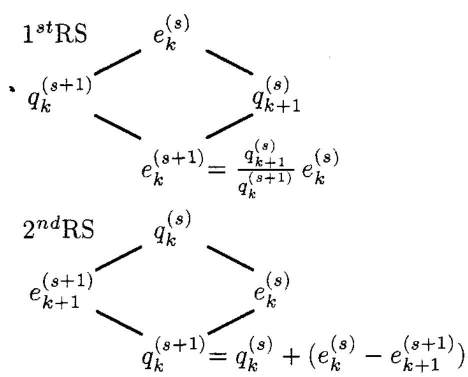
$$\begin{array}{ccc} f & \xrightarrow{\otimes} & a_f \\ \big\downarrow \mathcal{F} & & \big\uparrow \mathcal{F}^- \\ \mathcal{F}(f) & \xrightarrow{\times} & \left(\mathcal{F}(f)\right)^2 \end{array}$$

- Some matrix icons (Wilkinson, 1965):

 or $AL = LH$

 or $A = QR$

- Rhombus scheme (Schwarz, et al., 1972:166):



- Continued fractions:

$$1 + \mathop{\Phi}_{k=1}^{n} \frac{a_k}{b_k} \stackrel{\text{def}}{=} 1 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \ddots + \cfrac{a_{n-1}}{b_{n-1} + \cfrac{a_n}{b_n}}}}$$

with (space saving) variant notations:

$$= 1 + \frac{a_1|}{|b_1} + \frac{a_2|}{|b_2} + \cdots + \frac{a_n|}{|b_n}$$

$$= 1 + \frac{a_1}{b_1 +}\,\frac{a_2}{b_2 +}\cdots\frac{a_n}{b_n}$$

- Reduction to Hessenberg form via lower triangular similarity transformation (Wilkinson, 1965:357):

$$\overset{\text{A}}{\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}} \overset{\text{N}}{\begin{pmatrix} 1 & & \\ 0 & 1 & \\ 0 & \times & 1 \end{pmatrix}}$$

$$= \overset{\text{N}}{\begin{pmatrix} 1 & & \\ 0 & 1 & \\ 0 & \times & 1 \end{pmatrix}} \overset{\text{H}}{\begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}}$$

- Partitioning (Wilkinson, 1965:291):

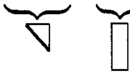
$$P_r = \left(\begin{array}{c|c} I_{n-r} & 0 \\ \hline 0 & I - 2v_r v_r^T \end{array}\right)$$

- Braces and matrices (Wilkinson, 1965:199):

$$\begin{array}{c} \\ p\left\{\begin{array}{c}\\ \\ \\ \\ \end{array}\right. \\ n-p\left\{\begin{array}{c}\\ \\ \\ \end{array}\right. \end{array} \begin{array}{c} \overbrace{\qquad\qquad}^{p} \quad \overbrace{\qquad\quad}^{n-p} \\ \begin{pmatrix} \times & \times & \times & \times & \times & \times & \times \\ 0 & \times & \times & \times & \times & \times & \times \\ 0 & 0 & \times & \times & \times & \times & \times \\ 0 & 0 & 0 & \times & \times & \times & \times \\ 0 & 0 & 0 & 0 & \times & \times & \times \\ 0 & 0 & 0 & 0 & \times & \times & \times \\ 0 & 0 & 0 & 0 & \times & \times & \times \end{pmatrix} \end{array}$$

- Matrices, braces, (dotted) partitioning and icons (space efficient variant):

$$\begin{array}{c} \\ p\left\{\begin{array}{c}\\ \\ \\ \\ \end{array}\right. \\ np\left\{\begin{array}{c}\\ \\ \\ \end{array}\right. \end{array} \begin{array}{c} \overbrace{\quad}^{p}\ \overbrace{\quad}^{n-p} \\ \left(\begin{array}{c} \times\times\times\times \vdots \times\times\times \\ 0\times\times\times \vdots \times\times\times \\ 0\,0\times\times \vdots \times\times\times \\ 0\,0\,0\times \vdots \times\times\times \\ \cdots\cdots\cdots\cdots \\ 0\,0\,0\,0 \vdots \times\times\times \\ 0\,0\,0\,0 \vdots \times\times\times \\ 0\,0\,0\,0 \vdots \times\times\times \end{array}\right) \\ \underbrace{\quad}\ \underbrace{\quad} \end{array}$$



The above examples resemble 'macho' behavior (showing off with TEX). I agree with that, but in practical situations I would like to use constructs which are as simple as possible.

What's Wrong, Doc?

Mathscripts differ from TEXscripts.

- The output[7]:

$$x = 1 + \left(\frac{y^2}{k+1}\right)^{1/3}.$$

looks different from:

```
$$x=1+\left({y^2\over k+1}
     \right)^{\!\!1/3}.$$
```

Because of this disparity, the problem is how to get a correct TEXscript, starting from just a mathscript. This is difficult because of the complexity of math typesetting, and the inherent complexity of TEX, if not because of the bewildering and confusing variety of TEX-based products.[8]

[7] Note that the kind of parentheses and the kind of division notation have to be specified as well.

[8] In this paper we restrict ourselves to plain TEX, and assume that no fancy, friendly, WYSIWYG user interface is available.

First, one has to find the appropriate format command from nearly a thousand.[9] In *The TeXbook* the following chapters are devoted to math formatting: 16 (11pp), 17 (21pp), 18 (23pp), 19 (14pp), 22 (242, ex22.9/11), 24 (up to 281, 15pp), 26 (5pp); Appendices A (33pp), B (6pp), F (13pp), G (7pp). Add to these the required general TeXnowledge of how to use TeX for non-complex documents and general page makeup, how to format tabular material (matrices, commutative diagrams), how to handle output routines, and how to use non-default fonts, and no-one would consider TeX to be trivial.[10]

Second, content and context-dependent extras have to be added, as demonstrated throughout this paper.

Third, once the TeX language is mastered, the difficulty of locating and correcting errors — misconceptions as well as typos — remains.[11] So add chapter 27 of *The TeXbook* to the above, just for completeness.

Once you have coped with everything that is mentioned above, you are still faced with true (La)TeX driver bugs and LaTeX's inconsistency. I was trapped by LaTeX's quote environment when I tried to get the opening quote to hang out. It did not work, not even after inserting `\null`.

Spivak (1986) has dealt with TeXing math in his delightful book, but alas, it is not a proper extension. My own perspective is to look for what is needed and to extend plain TeX in a compatible way, keeping overhead as low as possible. Plain TeX already provides enough TeXfalls.

The Bad News

The material in this section started as a list of pitfalls and grew into a general discussion with antidotes. (If readability for BLUe is reduced below par, I pitfailed.)

I would like to start by mentioning the nasty small white space on a new line after a heading. This creature can be killed by providing a comment symbol `%` immediately after the heading command (just a warm-up for the unwary[12]).

Too many. The 'too many' pitfall is a serious problem. It occurs when using many incompatible products which are partly, or not at all, understood.

In the typing project for which my assistance was asked, TeXed chapters showed different approaches: $\mathcal{A}_\mathcal{M}\mathcal{S}$-TeX was used in one, LaTeX in another, etc. This demonstrated the involvement of several typists and the lack of a common approach. The document also did not compile, showing that TeXing is one thing; getting it correct — if only just those braces — is quite another. This is especially true for typists not familiar with programming. Apart from the above, encoding was done inconsistently: $\mathcal{A}_\mathcal{M}\mathcal{S}$-TeX was used for some math symbols not available in plain, such as $\gtrless$. Commands like `\frac`, and `\overset` were used along with their plain functional equivalents. Obviously one typist was $\mathcal{A}_\mathcal{M}\mathcal{S}$-TeX oriented, while the others were not.

In short, the TeXscript was far from correct, suffered from too many tools, and otherwise was full of horrible TeXfalls. The Temme book didn't suffer from these TeXfalls, as it used plain TeX and only a few extra symbols.

I incurred the following problem when preparing this paper. This paper uses `ltugproc.sty`, and therefore LaTeX. In LaTeX `\eqalign`, etc. are not available, so I defined them. But, I did not think of redefining `\centering`, which has a different meaning within LaTeX than within plain TeX and as a consequence, `\eqalignno` did not produce the desired result. Another TeXfall was that `\eqalign` did not center in two-column format when `\eqno` was used as well! I had to first deactive the glue item '`\,`' of `\eqalign`. (For an explanation see *TB* 189.)

However, for all those mathematicians who practise self-publishing, it pays to encode as simply as possible.[13] Understanding the basics and adding a few macros will do, especially for those who otherwise have to rely on Word*whatever*. This is demonstrated by the Temme book, and as far as I understand it, it is also the attitude of the Grand Wizard himself.

[9] Cheswick (1990) has provided a KeyWord In Context with all the TeX and LaTeX commands. This is handy when in doubt as to whether a command is already in use.

[10] Beeton (1990) states that it was the intent of the $\mathcal{A}_\mathcal{M}\mathcal{S}$-TeX project to "simplify input of complex mathematical expressions."

[11] The TeXist's task has been silently increased by the parsing and correcting of the TeXscript in order to provide the author with proofs.

[12] This is overlooked in the Dutch course book on LaTeX, and also in the Dutch `brief` style, where the addressee label on the subsequent page headings is preceded by white space.

[13] This means that the tools should be powerful and mixing similar tools should be kept to a minimum.

On the other hand, I welcome the approaches taken by *TUGboat* (see Whitney and Beeton, 1989), and the AMS, (see the *AMS-TEX User's Guide*, 1990) where, on top of a common style, similar LaTeX and TeX procedural markup 'user interface' commands have been built. This provides freedom for authors to submit either LaTeX or plain TeX manuscripts, while the publisher can easily integrate these sources into one publication. This indicates that some TeXfalls can be taken care of by the publishing house.

Separation of concerns. *The* pitfall for author publishing, or self-publishing systems is insufficient awareness of the requirements in other fields. Not only does the author have to worry about the content, the organization, the power of the examples, the use and spelling of the language, and consistency, etc., but he also must worry about conventions of math typesetting, the computer system, and typing skills, not to mention proofreading.

First gains. The typographic markup pitfall reflects the temptation to specifically format how elements should *look*, instead of tagging the elements with the purpose of *identifying* them — a matter of abstraction and separation of concerns. One can think of the various headings: chapter, definition, theorem, and the like, where the formatting can be postponed and provided separately in style or format files. This pitfall can also be classified as the portability pitfall: submitting an article to another journal needs adaptation of the copy when descriptive markup is not used. I encountered the following in the TeXscript:

```
{\bf Summation of infinite
     series of complex functions.}
{\bf Theoretical background.}
\vskip1truecm
{\it1.2.1\underbar{Ten standard
            definitions}}.
\vskip1truecm
\underbar{Definition 1.}
```

As can be seen, a lot of typographical detail had been supplied. Agreed, it is generally available in the mathscript because authors are accustomed to supplying the section numbers and indicating bold and underlining. Using formats provided by the publisher does pay, because it is then the concern of the publisher to achieve the correct results.

To the same category of pitfalls belongs the typing of commands for creating extra white space along with each display, especially when the mathscript is full of crowded formulae. Instead of repeatedly typing `\vskip`'s, use can be made of the `\everydisplay` command, along with the assignment of new values to `\abovedisplayskip`, `\belowdisplayskip`, and their shorter variants.

Similarily, more white space can be placed between lines within a display. One does not have to modify the code because `\openup` is a cummulative command. Just say, for example, `\openup1\jot`, and interline spacing is increased by the given amount. AMS-TeX's `\spreadlines` is a disguise for this assignment, I presume.

Redefinition of existing commands is also a pitfall. First, trivially, because the original meaning of the command might be lost by accident. Second, when the command is to be customized — i.e., `\proclaim` *into* `\theorem` — one cannot simply use `\proclaim` *within* `\theorem` because `\proclaim` is an outer command. You must either remove `\outer` from `\proclaim` or copy the body of `\proclaim` into `\theorem`

Emptiness. The spacing pitfall is a difficult morass.[14] Once the automatic spacing is overruled by explicit spacing commands, the inconsistency pitfall opens up. A nice list of rules for spacing between symbols in math is given in Swanson (1986, chapter 3).

In math mode, spaces in the source file are ignored. Before and after each formula, space is inserted of size `\mathsurround`, which defaults to `0pt` in plain TeX (*TB* 162, 353). Within a formula the spacing is context-dependent, and determined by the class of the math character. Some symbols, those of the binary class for example, get extra spacing *around* them. Punctuation symbols have spacing *after* the symbol. The math character classes are given on page 154 of *The TeXbook*. For each class, the precise spacing values, related to the context, are given in the table on page 170 of *The TeXbook*.

Alas, in spite of — or because of — the above mentioned automatic mechanisms, extra space has to be inserted or deleted now and then.

- Some examples of additional positive and negative spacing in math mode:

$$(\lambda)_2\,{}_2F_1,\quad \int\!f(x)\,dx,\quad \Gamma_{\!2}+\Delta^{\!2},\quad \sum^\infty_{n=-\infty}\!\!\!\cos nt$$

are TeXed as (*TB* 168):

```
$$(\lambda)_2\,{}_2F_1,             \quad
        \int\!f(x)\,dx,             \quad
 \Gamma_{\!2}+\Delta^{\!2},         \quad
\sum^\infty_{n=-\infty}\!\!\!\cos nt$$
```

[14] Once in a while I think of TeX as dealing essentially with flexible spaces.

In the Temme book I found $n!n^2$ encoded as `$n!n^2$`, as opposed to $n!\,n^2$, which is correctly encoded as `$n!\,n^2$`.

Negative kerning after integral signs was not used either, especially with double integrals. The integral signs are spread too wide and stood too far away from the integrand. Summation symbols with large limits would also have benefitted from negative kerning.

Another aspect of spacing is ${}_1\phi_0(a;-;q,z,)$. The empty symbol ␣ could have been used by using `{\tt\char'040}`.

And what about placeholders? For example, the source `$\bigl(f,K_n(\cdot,y)\bigr)$` yields $\bigl(f,K_n(\cdot,y)\bigr)$? Introduce spaces around the placeholder via '`\,\cdot\,`'.

- Also of interest are expressions in exponents or indices. The Temme book contained:

$$e^{-z\sinh t+\nu t}$$

which does not look nice because of suppression of space around the operator. Introduce explicit thin spaces before and after the binary operator, or use parentheses around the argument of the function. Why not format $\exp(-z\sinh t+\nu t)$, in agreement with the general advice to use `\exp` for non-simple exponents? For other situations where `\exp` cannot be used, `\hbox{$...$}` can be considered as a sub- or superscript, yielding the correct spacing.[15]

In the TEXscript I encountered:

```
\wit  %meaning white space
$$|t| \quad < \quad | \quad x \quad -
  \quad (x + 1)^{\frac12}
  \quad (x - 1)^{\frac12} \quad |,$$
\wit
```

Spacing between formulae was not understood and done inconsistently. Unnecessary extra white space was introduced in too many places by the insertion of hundreds of `\,`, `\quad`, and `\qquad`'s.

On the use of `\(q)quad`, the best quote may be found on page 166 of *TB*:

> The traditional hot-metal technology for printing has led to some ingrained standards for situations like this, based on what printers call a "quad" of space. Since these standards seem to work well in practice, TEX makes it easy for you to continue the tradition: When you type '`\quad`' in plain TEX format, you get a printer's quad of space in the horizontal direction. Similarly, '`\qquad`' gives you a double quad (twice as much); this is the normal spacing for situations like the F_n example above.

A little further down on page 166 of *The TEXbook* the reader's attention is drawn to a different approach which is needed in alternating math and text in a paragraph.

`$F_n=F_{n-1}+F_{n-2}$, for $n\ge2$.`

Consistency can be enhanced by defining a document element, and subsequently using the element via its name. For example, the real part of z can be obtained in math mode via `\Re z`, once we have defined

`\def\Re#1{{\rm Re}\,#1}`

In the Temme book this was implemented via `{{\cal R}\,#1}`, which is especially handy when real parts of quantities are used in formulae. In the TEXscript I also encountered the following subtle examples which, after correction, read

$$C_\nu^\lambda(-z)=\cos\pi\nu\,C_\nu^\lambda(z)-\sin\pi\nu\,D_\nu^\lambda(z),$$

where '`\,`' (extra space) had to be inserted after the arguments of the trigonometric functions. In the Temme book, similar situations were circumvented via parentheses, $\cos(\pi\nu)C$, via `$\cos(\pi\nu)C$`; no extra space had to be inserted after the closing parentheses (*TB* 170).

Class unawareness. Several examples are provided below which demonstrate the unawareness of mathematical characters belonging to one of the eight classes, (*TB* 154).

- An example on page 171 of *TB* shows:

  `$|-x|$`, `$\left|-x\right|$`,
  and `$\lfloor-x\rfloor$`

  with the results $|-x|$, $\left|-x\right|$, and $\lfloor-x\rfloor$.[16] In the Temme book I found $\gamma^\star(a,x)$, as well as $\gamma\star(a,x)$. Do you see the difference?

Innocent braces. The pitfall here is that braces are not harmless but yield a formula of class 0 within math mode!

- Compare the following results:

  $a+b$, $a{+}b$ and a+b

  with their respective source code:

  `$a+b$` `$a{+}b$` `a+b`

[15] Petrycki (1991) also mentions difficulties with math spacing in TEX. The spacing around growing parentheses and the lack of spacing in sub- and superscripts is unacceptable.

[16] Why $|-x|$, and not just $|x|$? Furthermore, norm fences don't belong to the opening or closing class.

The first + is of the class binary and takes spacing according to the table on page 170 of *TB*, and in the second, the + is reduced to the class zero, and takes only `\mathsurround` spacing (default in plain TEX is 0pt).

- Analogously:

  $a = b\,,\qquad a{=}b\qquad$ a=b

  are obtained via:

  `$a=b$     $a{=}b$     a=b`

TEXperts frequently use braces, especially in alignments where empty formulae are to be used now and then. Another occurrence of *harmful* braces is given by `\cnt={1}` and the like, yielding an error message.

The general issue is that in math mode braces have the extra function of creating a subformula — not only delineating a scope — with the resulting subformula being an atom of class ordinary (*TB* 154, 158, etc.), and therefore BLUe must understand the various atom classes.

Whoops. What about the following?

```
\def\Inn{{\raisebox{1pt}
       {{\hbox{{$\in$}}}}}}
```

The concept of a binary operator was not accounted for, yielding the wrong spacing. TEX did not know that the raised `\in` had to be considered as an operator and therefore it was reduced to class ordinary, taking `\mathsurround` spacing.

- In the Temme book, I found:

  $$2\pi i\,{\rm Res}_{s=e^{i\pi}}f(s)=-2\pi i\,e^{i\pi z}$$

  TEXed via:

  ```
  2\pi i\, {\rm Res}_{s=e^{i\pi}}f(s)
      =-2\pi i\, e^{i\pi z}.
  ```

- I prefer the `Res` operator (in display and in agreement with Swanson, 1986) to look like this:

  $$\mathop{\rm Res}_{s=e^{i\pi}} f(s)=-e^{i\pi z}$$

  encoded as:

  ```
  \mathop{{\rm Res}}_{s=e^{i\pi}}
         f(s)=-e^{i\pi z}
  ```

An example of spacing which has to be suppressed is in *<name>*, (coded as `${<}name{>}$`). The relational operators are not used as such, and are forced by the braces into class ordinary. The latter example is taken from the BNF notation of programming languages, denoting meta-linguistic variables. (DEK uses ⟨ and ⟩ in his syntax, with similar use as (and).)

Just a comma. The number 3,14 innocently encoded as `$3,14$`, would yield $3,14$. The correct formatting is `$3{,}14$` (*TB* 134). Braces are needed again. The comma belongs to the punctuation class of math symbols and the surrounding braces — creating a subformula — reduce it to class ordinary, which requires no extra spacing. As part of the text, the number could have been obtained via `3,14`, with no `$`'s around it.

Binary operator vs. punctuation mark. A dot is used for a (binary) multiplication operator and as a punctuation mark. Three dots in a row don't yield an ellipsis. The formatting of the ellipsis is context-dependent: they can be at the axis of the formula, or at the baseline. Moreover, they can be vertical, or diagonal.

Multiplication in mathematics can be denoted by: $a\times b$, $a\cdot b$, or implicitly by a thin space $a\,b$, which has to be marked up explicitly. If you simply enter `a b`, the space will be gobbled up by TEX when it's in a good mood. Typists, and those used to the old typewriter, err by using '`x`' for `\times`, and by using the punctuation dot '`.`' instead of `\cdot` (with the binary multiplicator operator raised above the baseline).[17]

The real issue is that the handwritten symbol must be recognized from the context: is it a punctuation mark, an operator, or a significant space?

Colons: is there a difficulty? A colon as a *punctuation* symbol can be obtained via the `\colon` command, and as a *relation* symbol via '`:`', (*TB* 134).

- Examples of colons:

  $$f\colon A\to B,\quad \{x:x>5\}$$

  are obtained via:

  `$$f\colon A\to B,\quad \{x:x>5\}$$`

The Temme book used ':' throughout.

CMR fonts. Text in displays and standard function names traditionally use roman fonts, Swanson (1986, Table IV).

Sinus hyperbolicus ($\sinh x$) was encoded as `$\hbox{sin }hx$`, demonstrating bad handwriting by the author, and incorrect encoding. The TEXist was not familiar with the hyperbolic function names and therefore could not compensate for the bad handwriting. I also encountered the following badly encoded examples:

[17] To this category of misuses I also include 1 vs. l, 0 vs. o, U vs. ∪ etc. For more examples of these erroreous similarities, see "A Manual for Authors of Mathematical Papers" (AMS, 1973).

```
\cos\,\alpha
h^\lambda_\nu (z) \, : = _2F_1( ... )
\hbox{ for Re }\, z \, > \, 0
```

Chameleons. Regarding the chameleon pitfall, I mention those situations where TEX can't determine the correct sizing from the context. TEX provides facilities for automatically formatting the right size, given the context. TEX provides, for example, the correct sized openings and closings for a matrix, when these are specified by `\left...` and `\right`.[18] A TEXfall occurs when the context does not prompt for the possible need for another size (while BLUe expects TEX to do everything correctly).

- An example of context-dependent sizes, as inspired by Spivak (1986:55):

```
$$||\alpha(\sqrt a+\sqrt b)||
    \leq|\alpha|.
  (||\sqrt a+\sqrt b||).$$
```

  yields the result:

$$||\alpha(\sqrt{a}+\sqrt{b})|| \leq |\alpha|.(||\sqrt{a}+\sqrt{b}||).$$

- Better encoding would be:

```
$$\bigl\|\,\alpha(\sqrt{\mathstrut a}+
      \sqrt{\mathstrut b}\,)\,\bigr\|
  \leq|\alpha|\,
  \bigl\|\sqrt{\mathstrut a}+
   \sqrt{\mathstrut b}\,\bigr\|$$
```

  with resulting:

$$\bigl\|\,\alpha(\sqrt{\mathstrut a}+\sqrt{\mathstrut b}\,)\,\bigr\| \leq|\alpha|\,\bigl\|\sqrt{\mathstrut a}+\sqrt{\mathstrut b}\,\bigr\|$$

In this example, the norm fences are made larger and all `\sqrts` have been told to have arguments of `\mathstrut` size (Ascender and descender invariance!). Moreover, the multiplication dot can be replaced by a thin space.

- Another use of the vertical bar occurs in set notation (*TB* p175, ex18.21):

$$\bigl\{\,x^3\bigm|h(x)\in\{-1,0,+1\}\,\bigr\}.$$

  obtained via:

```
$$\bigl\{\,x^3\bigm|h(x)\in%
      \{-1,0,+1\}\,\bigr\}.$$
```

This not only demonstrates the correct size of the outer braces and the vertical bar, but also exhibits awareness of the binary operator function of the vertical bar, with the appropriate spacing by default. Set notations in the Temme book had not been marked up via the use of `\mid|` or its variants. For nested parentheses the `\big`, etc., representations were not used. The old technique with square brackets for the outer parentheses was used: $[\ln(z+1)]^m$.

Note that it looks better to introduce some spacing along with the outer braces. If an author wants these kinds of results he has to indicate that in the mathscript. I expect these kinds of issues will not be touched upon or will be handled inconsistently in a document of nontrivial size.

[18] TEX automatically adapts the correct sizing for `(` or `\{` , when using `\left` or `\right`.

Triads. The 'three dots in a row,' or ellipsis, are heavily used in mathematical notation.

- For example:

$$x_1+\cdots+x_n, \quad x_1x_2\ldots x_n$$

  is obtained by using the `\cdots` and `\ldots` commands.

The issue is not to type '...', but to use the `\cdots` or `\ldots` command, respectively.

When the ellipsis is followed by a punctuation dot, a small extra space '`\,`' has to be specified: `1+x+x^2+\cdots\,.` An ellipsis is often used in a fixed context.

- For example: for $i=1$, 2, $\ldots\,$, n. can be obtained via:

```
for $i=1$,~2, $\ldots\,$,~$n$.
```

Such sentences are candidates for abbreviation into a macro such as `\for in`:

```
\def\for#1#2{
   for $#1=1$,~2, $\ldots\,$,~$#2$},
```

maintaining consistency. It also reduces the number of keystrokes. Note that `\dots` is not substituted for `\ldots\,`, and '`\,`' is needed. The pitfall of confusing the use of `\dots` and `\ldots` is also circumvented by the use of the `\for` abbreviation. The Temme book was inconsistent in the use of `\cdots` and `\ldots`.

In order to facilitate looking up shortcuts, Wichura (1990) has provided some macros which yield a table consisting of a math-writing-column and a corresponding TEX-input-column — a fancy tool suited for typists. This is not enough to solve the typist's problems, though it might help.

Real life. Other dots are also used: vertical dots in matrices, (*TB* 177), and diagonal dots, (*TB* 177, ex18.45). Triple-dotted letters are particularly captivating.

With respect to the continued fraction example provided in the 'Am I BLUe' section, Swanson (1986) just uses an ellipsis. Her variant notations also differ somewhat. Those given here originate from Peter Henrici (1977). His Φ symbol is the absolute space saver.

The auxiliary symbol `\cf`, the Φ, must be made robust, so that it can be used with other styles as well, yielding appropriate sizes. TEX provides \mathchoice for this purpose. For example, \cf should have been defined as:

```
\def\cf{\mathop{\mathchoice{%
\grkop\Phi}{%Magnified
\Phi}{\Phi}{\Phi}       }}
```

Accents differ. Accents are treated differently in math mode than in horizontal mode. DEK decided to provide different math mode accent commands (*TB* 135). It is unclear to me why the commands have not been overloaded.

Embellishments.

- We have barred letters:

$$\bar z,\ \overline z,\ \bar P,\ \overline P,\ \bar h,\ \hbar,\ \overline{AB}$$

obtained via:[19]

```
$$\bar z,\ \overline z,\
\bar P,\ \overline P,\
\bar h,\ \hbar,\
\overline{AB}$$
```

It is easy to forget that \bar provides a bar of fixed length, and looks strange when used over capitals. It is wrong to use it with subformulae in general; this holds for all accents. The Temme book used \bar over capitals. Note also the use of \vec a, for $\vec a$, \vec A, for $\vec A$, (A accented with an arrow), and \overrightarrow A, for $\overrightarrow A$, or \overrightarrow{AB}, for $\overrightarrow{AB}$, (see *TB* 136, 359) — once again a source of confusion and inconsistency. Swanson (1986) advises the use of boldface characters for vector notation.

Normalized functions are often denoted by dotted letters, and because a dot is a tiny blot of ink, more pronounced dots are needed. I encountered the use of bulleted letters, which look awful. Something like a bold dot is needed. Bold dotted P — $\dot P$ — and simple dotted P — $\dot P$ — are obtained via \bdot P and \dot P, with the use of:

```
\def\bdot#1{{\bf\dot{{\mit#1}}}}
```

Note the extra pair of braces, another TEXfall.[20] The Temme book even used triple-dotted letters, to denote the Schwarz derivative, $\dddot x$. The above was done via:[21]

```
\def\dddot#1{%
\setbox0=\hbox{$#1$}
\vbox to\baselineskip{\vss\hbox{%
 ${\mkern1mu.\mkern-2mu.\mkern-2mu.}
              \phantom{\char'177}$}
      \kern-ht0
      \copy0}}%end dddot
```

Alas, we both committed a TEXfall. The above approach is based upon rule 12 in Appendix G-12 page 443 of *The TEXbook*, and should be compatible with \dot, and \ddot. It is not compatible at the moment and Appendix G-12 should be revisited. (My TEXnigma could not reproduce Malcolm's results when fed his experimentally found kernings.) In math literature I found $\ddot y$.

Prime-ry. Characters are primed in math mode to denote derivatives. For instance, y' is encoded by y^\prime or via the more natural shorthand y'. The ' character is active and overloaded (context-dependent).[22] The TEXfall is that symbols which have limits can't be simply primed in display.

- Compare the following examples:

$$\sum' \quad {\sum}' \quad \mathop{\sum'}^n_{k=0} \quad \mathop{{\sum^n_{k=0}}'} \quad \mathop{\sum\mathstrut'}^n_{k=0} \quad \mathop{{\sum}'}^n_{k=0} \quad \mathop{{\sum}'}_{k=0}^n$$

obtained via:

```
\def\acclap#1{
   \raise\hgtsig\hbox to0pt{$#1$\hss}}
\newdimen\hgtsig
\setbox0=\hbox{$\displaystyle{\sum}$}
\hgtsig=\ht0\relax
\advance\hgtsig by -1.75ex
$$\sum'                          \quad
{\sum}'                          \quad
\mathop{\sum'}^n_{k=0}           \quad
\mathop{{\sum^n_{k=0}}'}         \quad
\mathop{\sum\mathstrut'}^n_{k=0}\quad
\mathop{{\sum}'}^n_{k=0}         \quad
\mathop{{\sum}\acclap'}_{k=0}^n $$
```

DEK's solution is given in ex18.44 of *The TEXbook*. Primed summation symbols are used in Chebyshev expansions, for example.

The double primed summation symbol can be obtained in a similar fashion. As mentioned by Knuth, the problem is to center the lower limit with respect to the summation symbol proper.[23]

$\frac{1}{2}$'s never have the right size. BLUe invariably goes wrong when TEXing 'halves.'

[19] AMS fonts also provides \hslash similar to \hbar but with the bar inclined.

[20] The extra braces are necessary in order to keep the font changes local.

[21] Clark (1987) used a fixed vertical kerning and therefore his code is not robust.

[22] The accent itself can be obtained within math mode via \mathchar"'.

[23] Scripting the primed operator would violate this.

- The following example:

$$D^\lambda_0(z)=4a_\lambda\, z\, {}_2F_1(\textstyle\lambda+{1\over2},{1\over2};{3\over2};z)$$

  is TEXed via:

```
$$D^\lambda_0(z)=4a_\lambda\, z\,
{}_2F_1(\textstyle\lamda+{1\over2},
        {1\over2};{3\over2};z)$$
```

In the Temme book, I encountered the above notation, and also $F(1/2,1/2;3/2;z^2)$. Later, I stumbled upon $\int^{\frac12\pi}$ along with the more usual $\int^{\pi/2}$. The latter is also recommended by Swanson (1986).

- In the TEXscript,[24] I found:

$$D^\lambda_0=-{\textstyle\sin{\pi\nu\over2}\, C^{\scriptscriptstyle\lambda\over\scriptscriptstyle2}_{\kern-1pt{\scriptscriptstyle\nu\over\scriptscriptstyle2}}}$$

  via:

```
$$D^\lambda_0=-{\textstyle
\sin{\pi\nu\over2}\,
C^{\scriptscriptstyle\lambda\over
              \scriptscriptstyle2}
 _{\kern-1pt{\scriptscriptstyle\nu\over
              \scriptscriptstyle2}}}$$
```

The general point is to kern and force the right style. Another example of where the right style is coerced occurs when the summation symbol takes stacked limits. Explicit mentioning of `\scriptstyle` in both operands of the `\atop` command is needed (*TB* 145).

Knuth (1985), mentions the use of a typographer's '$^1/_2$,' especially in recipes, which works better than a mathematician's '$\frac12$'.

Various O0$O\emptyset\circ o$'s.

Mathscript O's are overloaded: '$\emptyset$' (the empty set), $f\circ g\colon x\mapsto f\bigl(g(x)\bigr)$ (composition), and the order symbols $o(h^2)$ and $O(h^2)$:

- `$\emptyset$`,
  `$f\circ g\colon`
  `  x\mapsto f\bigl(g(x)\bigr)$`,
  `$o(h^2)$`,
  `$O(h^2)$`.

We also have trigonometric and temperature degrees 30° and °K (*TB* 180). Another challenge is a notation for the zero vector, (see *TB* ex18.6).

Backslash penances. Because of the special function of the backslash, people are in trouble when the symbol itself is wanted. In horizontal mode the backslash, as such, can be obtained by selecting the symbol from the `tt` font, (*TB* 429) position '134 (decimal 92), via `{\tt \char'134}`. In math, the backslash is used for the setminus (binary) operator and for denoting cosets; the latter requires no space. Compare:

$$A\setminus A=\emptyset \quad\text{and the cosets of } G \text{ by } H\text{: } G\backslash H$$

TEXed by use of `\setminus` and `\backslash` (*TB* 436). Needless to say, the mathscript contained several setminus operations, while in the TEXscript the `\backslash` was used throughout.

Over and over. BLUe is encouraged to treat a fraction as a subformula (*The TEXbook* 140, ex17.3), and to use braces around <*formula* `\over` *formula*> — a good habit to adhere to throughout. I was trapped when changing `\left(` and `\right)` into `\bigl(` and `\bigr)`. The former notation creates a subformula while the latter does not — this is not robustness!.

Swanson (1986) advises us to consider that the use of slashes when saving space can be achieved while preserving clarity of exposition.

In `\buildrel` (*TB* 437), `\over` is overloaded.

Too difficult. Hypergeometric functions sometimes take 'matrices' as arguments. As stated in *TB* (page 178), the use of `\(p)matrix` in the text of a paragraph yields results which are too big: $M_n(z)={}_{n+1}F_n\bigl({k+a_0,\atop}{k+a_1,\dots,k+a_n\atop k+c_1,\dots,k+c_n};z\bigr)$ is obtained via:

```
$M_n(z)={}_{n+1}F_n
  \bigl({k+a_0,\atop\phantom{kc_1}}
        {k+a_1,\dots,k+a_n\atop
             k+c_1,\dots,k+c_n};z\bigr)$
```

Note the automatic centering 'on the axis' of the last argument. A fuzzy issue involves what to do with empty arguments, especially when several `\atop`'s are used in a row. The general approach is to use `\mathstrut`s. For two `\atop`s the use of `\phantom` will yield aligned results, as demonstrated in the given example.

The late Yudell Luke used the '|' symbol instead of ';'. For example:

$${}_pF_q\Bigl(\,{\alpha_p\atop\rho_q}\mathpunct{\bigm|}\,z\Bigr)={\Gamma(\rho_q)\over\Gamma(\alpha_p)}\,G^{1,p}_{p,q+1}\Bigl(-z\,\mathpunct{\bigm|}\,{1-\alpha_p\atop0,1-\rho_q}\Bigr)$$

is obtained via:

```
$${}_pF_q
\Bigl(\,{\alpha_p\atop\rho_q}
  \mathpunct{\bigm|}\,z
\Bigr)=
{\Gamma(\rho_q)\over
  \Gamma(\alpha_p)}\,
G^{1,p}_{p,q+1}
\Bigl(-z\,\mathpunct{\bigm|}\,
  {1-\alpha_p\atop0,1-\rho_q}
\Bigr)$$
```

[24] To be avoided, (Swanson, 1986).

Here the vertical bar is coerced into serving as a punctuation symbol, with some extra spacing added. Also note the lack of spacing in the subscripts and superscripts (*TB* 170).

The TEXscript contained:

```
\begin{dispeqs}
M_n(z):={}_{n+1}F_n
\left(\aligned
k+a_0,&k+a_1,\dots,k+a_n;z\\
      &k+c_1,\dots,k+c_n
      \endaligned\right)
\tag1.2.55
\end{dispeqs}
```

which does not reflect the centering of '; z.' It also demonstrates the use of an unnecesssary 'non-standard' environment, inhibiting intelligibility. The desired result was not obtained, while the TEXscript was encoded with too much, and unnecessary, complexity. (Fortunately, I have never needed to talk about hypergeometric functions by phone.)

Jam Session I

The conclusion of this work is that TEXing math is too difficult for non-TEX-trained typists. The need for more TEX instruction must be taken into consideration by mathematicians as well as those who keyboard the TEXscripts. Both the author and the typist need to have more than a nodding acquaintance with TEX. Education is needed and discipline must to be adhered to. What about a discipline of TEXing?

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Rhapsody in Blue

The following is the source code for the 'Am I Blue' section.

```
\bi
\item Selections from chapters
16\dash 18 in {\sl \TB\/}:
%
$${\bf S^{\rm-1}TS=dg}(\lambda_1,
   \ldots\,,\lambda_n)=\bf\Lambda$$

$$\sum_{k=1}^{\infty}{1\over2^k}=1,\quad
  \sqrt{1+\sqrt{1+\sqrt{1+x}}}$$

$${f(x+\Delta x)-f(x)\over\Delta x}\to
  f'(x)\quad{\rm as}\quad\Delta x\to0$$
%
$$\{\underbrace{\overbrace{\mathstrut a,
  \ldots,a}^{k\;a\mathchar''\rm s},
    \overbrace{\mathstrut b,\ldots,b}
      ^{l\;b\mathchar''\rm s}}
      _{k+l\rm\;elements}\},
\quad
 2\uparrow\uparrow k\mathrel{\mathop=
  ^{\rm def}}
  2^{2^{2^{\cdot^{\cdot^{\cdot^2}}}}}
  \vbox{\hbox{$\Big\}\scriptstyle k$}
  \kern0pt}
$$
%
\item {The Cardano solution of $x^3+px=q$,
with $p,q\ge0$}  reads:
\ei
$$ \root3\of{\sqrt{p^3/27-q^2/4}+q/2}-
   \root3\of{\sqrt{p^3/27+q^2/4}-q/2}$$
\bi
\item {Derivatives}:\footnote{Kerning
an extra point in superscripts was
pointed out by Daniel Olson.}\\
$\dot y\,\ddot y\, \dot{\ddot y}\quad
  y'\,y''\,y'''                 \quad
\partial_xy\,\partial_x^{\kern1pt2}y,
\partial_x^{\kern1pt3}y$
%

%Some more from analysis
\item {Bessel equation}:
$$z^2w''+zw'+(z^2-\nu^2)w=0$$
with solutions:
\par\noindent
$J_{\pm\nu}(z)$, Bessel function of the
first kind,\par\noindent
$Y_{\pm\nu}(z)$, Bessel function of the
second kind (Weber),\par\noindent
$H_\nu^{(1)}(z)$, and $H_\nu^{(2)}(z)$,
Bessel function of the third kind
(Hankel).
%
\item {Primed summation symbols} are used
in Chebyshev expansions:
\def\acclap#1{
   \raise\hgtsig\hbox to0pt{$#1$\hss}}
\newdimen\hgtsig
\setbox0=\hbox{$\displaystyle{\sum}$}
\hgtsig=\ht0\relax
\advance\hgtsig by -1.75ex
$$\displaylines{\quad
 \setbox0=\hbox{$\displaystyle
 \mathop{{\sum}\acclap'}_{k=0}^n$}
 \dp0=0pt \box0    %Neglect dp size
 a_kT_k(x)\mathrel{\mathop=^{\rm def}}
 {\textstyle.5}\kern1pt a_0+a_1 x+
   a_2T_2(x)+\cdots\hfill        \cr
   \hfill{}+a_nT_n(x).\quad      \cr
}$$
%
\item {Hypergeometric function}:
$$M_n(z)={}_{n+1}F_n\Bigl({k+a_0,
   \atop\phantom{kc_1}}
   {k+a_1,\dots,k+a_n
   \atop k+c_1,\dots,k+c_n};z\Bigr)
$$
%
\item {From Swanson (1986:40)}:
$$\displaylines{
  \int\nolimits_U\delta(I)\mu(I)
                 \leq{}\hfill\cr
{}\sum_{{\cal D}}
         \sum_{{\cal D}_{I'}}
  \biggl[\int\nolimits_J
  \alpha(J')\mu(J')-\alpha(J)\mu(J)
                        \hfill\cr
%CGL278 next lay-out modified
  \qquad\hfill  {}-\int\nolimits_J
    [\{s(\alpha\eta)(J')\}
       /\eta(J')]\mu(J')\biggr]\cr
{}+\biggl[ %CGL278 modified, to align
%                   with double sum
       \sum_{{\cal D}}
       \sum_{{\cal D}_{I'}}
   |\alpha(J)-[\{s(\alpha\eta)(J)\}
   /\eta(J)]|\mu(J)\biggr] \hfill\cr
  \hfill
   {}\times\biggl[
       \sum_{{\cal D}}
       \sum_{{\cal D}_{I'}}
   |\alpha(J)-[\{s(\alpha\eta)(J)\}
   /\eta(J)]|\eta(J)\biggr] \cr} $$
%
%
\item {Magic squares}
(D\"urer's 4-by-4  with dotted lines):
$$
\vcenter{\tabskip0pt\offinterlineskip
 \hrule\halign{\strut\vrule height3ex
    depth1.5ex\relax
    \enspace\hfil#\hfil\enspace\vrule&&
           \strut
```

```
    \enspace\hfil#\hfil\enspace\vrule\cr
    2& 7& 6\cr \noalign{\hrule}
    9& 5& 1\cr \noalign{\hrule}
    4& 3& 8\cr}              \hrule}
\quad
%
\def\cdotfill{\cleaders\hbox{%
$\mathsurround0pt\mkern1.5mu{\cdot}
 \mkern1.5mu$}\hfill}%end cdotfill
\def\vdotfill{%
   $\vcenter{%dotted line
   \vbox to5ex{\cleaders\hbox{%
   $\mkern1.5mu{\cdot}\mkern1.5mu$}
    \vfil}}$\hfil}%end vdotfill
%
\vcenter{\tabskip0pt
 \offinterlineskip
 %\hrule replaced by \cdotfill
 \halign{
   \vdotfill
   $\vcenter{\hbox to 5ex{\hss\oldstyle#\hss}
    }$%\hfil\enspace
   \vdotfill&&      %repetition
   %\enspace\hfil
   $\vcenter{\hbox to 5ex{\hss\oldstyle#\hss}
    }$\hfil\enspace
   \vdotfill  \cr%end template
   \multispan4\cdotfill\cr
   16& 3& 2&13\cr
   \multispan4\cdotfill\cr
    5&10&11& 8\cr
   \multispan4\cdotfill\cr
    9& 6& 7&12\cr
   \multispan4\cdotfill\cr
    4&15&14& 1\cr
%No bold oldstyle 1514 available
   \multispan4\cdotfill\cr
}%end \halign
}$$
%
\item{Calculation flow of autocorrelation
function
$a_f$} (inspired by {\sl TB\/} 358, ex18.46).
${\cal F}$ denotes Fourier transform and
${\cal F}\strut^{-}$ the inverse Fourier
transform:
%
\def\lllongrightarrow{\relbar\joinrel%
       \relbar\joinrel\relbar\joinrel%
       \relbar\joinrel\rightarrow}
\def\llongrightarrow{\relbar\joinrel%
        \relbar\joinrel\rightarrow}
\def\normalbaselines{%
            \baselineskip20pt
            \lineskip3pt
            \lineskiplimit3pt}
\def\mapright#1{\smash{\mathop{
   \llongrightarrow}\limits^{#1}}}
\def\lmapright#1{\smash{\mathop{
   \lllongrightarrow}\limits^{#1}}}
\def\mapdown#1{\Big\downarrow
      \rlap{$\vcenter{\hbox{$#1$}}$}}
\def\mapup#1{\Big\uparrow
      \rlap{$\vcenter{\hbox{$#1$}}$}}
$$%Diagram
\matrix{f&\lmapright\otimes&a_f\cr
    \mapdown{{\cal F}}&&\mapup{%
          {\cal F}\strut^{-}}\cr
    \hbox to 0pt{\hss${\cal F}(f)$\hss}
    &\mapright\times\hfil&
    \hbox to 0pt{\hss$\bigl({\cal F}(f)
                   \bigr)^2$\hss}\cr}
$$
%
%%%%%%%%  Amy's Diagonal line %%%%%%
\newif\ifminuswd
\newif\ifminusht
\newbox\dotwide
\newbox\dotbox
\newdimen\savehtofdiagline
\newdimen\htofdiagline
\newdimen\wdofdiagline
\newdimen\dotmove
\newdimen\newsaveht
\newdimen\savewdofdiagline
%
\def\diagline #1 #2 wd #3 ht{%
%Amy Hendrickson, improved
%version of TB 6#2, 1985
\global\setbox\dotwide=\hbox{#1}%
\global\setbox\dotbox=
   \hbox to0pt{\hss#1\hss}%
\global\wdofdiagline=#2\relax%
\ifdim\wdofdiagline<1sp
   \global\minuswdtrue
   \global\advance\wdofdiagline
                by-2\wdofdiagline
\else\global\minuswdfalse%Robustness
\fi% turn neg dim to positive
\global\htofdiagline=#3\relax%
\ifdim\htofdiagline<1sp
   \global\advance\htofdiagline
   by-2\htofdiagline\global\minushttrue
\else\global\minushtfalse%Robustness
\fi
\global\dotmove=1pt%%
\setbox1=\hbox{%
   \global\divide\htofdiagline
    by\the\wdofdiagline\relax}%
\ifminuswd\rlap\bgroup%
\else\bgroup\fi%
\loop\ifdim\htofdiagline>.4pt%CGL mod
\global\divide\htofdiagline by2%
\global\divide\dotmove by2\repeat%
\global\savehtofdiagline=\htofdiagline%
\loop\ifdim\wdofdiagline>0pt%
\hskip\ifminuswd-\dotmove%
      \else\dotmove\fi%
```

```
\ifminusht\lower\else\raise\fi%
      \htofdiagline\copy\dotbox%
\global\advance\htofdiagline
          by\savehtofdiagline%
\global\advance\wdofdiagline
          by-\dotmove\repeat%
\egroup%
}% end diagonal line
%
\def\icmat#1#2{%ICon MATrix(rectangular)
%#1 is ht of icon matrix, e.g. 4ex
%#2 is wd of icon matrix, e.g. 2ex
\vbox to#1{\hrule
           \hbox to#2{\vrule height#1
                         \hfil\vrule}
           \hrule}
}%end icmat
%
\def\icurt#1#2{%IConUpperRightTriangle
%#1 is ht of icon matrix,
%with UT the upper triangular part,
%e.g. 4ex #2 is wd of icon
%(upper triangular) matrix, e.g. 2ex
\vbox to #1{\hrule
   \hbox{\diagline . #2 wd -#2 ht\vrule}%
   \vfil}%
}%end icurt
%
\def\iclt#1#2{%IConLowerLeftTriangle
%#1 is ht of icon matrix,
%with LT the upper triangular part,
%e.g. 4ex #2 is wd of icon
%(lower triangular) matrix, e.g. 2ex
\vbox to #1{\vfil
   \hbox{\vrule\diagline . #2 wd -#2 ht}%
   \hrule}%
}%end icllt
%
\def\icuh#1#2#3{%IConUpperHessenberg
%#1 is size of icon matrix,
%with UH the upper Hessenberg part, e.g. 4ex
%#2 is wd of icon (upper Hesenberg)
%matrix, e.g. 1ex
%#3 is size Lower Left triangular part, #1-#2
%(for simplicity the latter is added,
% could have been calculated, perhaps some
% inconsistency test could be incorporated)
\vbox to #1{\offinterlineskip\hrule%
   \hbox to#1{\vrule height#2 depth0pt
                \relax\hfil\vrule}%
   \hbox to#1{\diagline . #3 wd -#3 ht
             \hfil\vrule}%
   \hbox to#1{\hfil \vrule width#2
                           height.2pt}%
   }%
}%end icuh
%
\item {Some matrix icons} (Wilkinson, 1965):
$$\eqalign{
\vcenter{\icmat{4ex}{4ex}}\kern2ex
   \vcenter{\icllt{4ex}{4ex}}
&=\kern1ex
   \vcenter{\icllt{4ex}{4ex}}
   \vcenter{\icuh{4ex}{1ex}{3ex}}
&\quad{\rm or} \quad AL=LH \cr
\noalign{\vskip2ex}
\vcenter{\icmat{6ex}{3ex}}\kern1ex
&=\kern1ex\vcenter{\icmat{6ex}{3ex}}
  \kern1ex\vcenter{\icurt{6ex}{3ex}}
&\quad{\rm or} \quad A=QR\cr
}$$

%Rhombus scheme
\newbox\ru %
\newbox\rl %
\setbox\ru=\hbox{%
   \diagline . 4ex wd +2ex ht\relax}%
\setbox\rl=\hbox{%
   \diagline . 4ex wd -2ex ht\relax}%
%
\item {Rhombus scheme}
(Schwarz, et al., 1972:166):

$$\displaylines{%\indent
\vbox{\offinterlineskip\tabskip=0pt
\halign{\hfil$#$%left element
&\hfil$\vcenter{#}$\hfil%left lines
&\hfil$#$\hfil          %middle elements
&\hfil$\vcenter{#}$\hfil%right lines
&$#$\hfil               %right elements
\cr                     %end template
1^{st}{\rm RS}\hfill
        & &e^{(s)}_k&&&     \cr
&\copy\ru& &\copy\rl&       \cr
q^{(s+1)}_k&&&&&q^{(s)}_{k+1}\cr
&\copy\rl& &\copy\ru&       \cr
&     &e^{(s+1)}_k
   &\omit$={q^{(s)}_{k+1}\over q^{(s
+1)}_k}\,e^{(s)}_k$\hfil\hidewidth\cr
\noalign{\vskip1ex}
2^{nd}{\rm RS}\hfill
        & &q^{(s)}_k&&     \cr
&\copy\ru& &\copy\rl&       \cr
e^{(s+1)}_{k+1}&&&&e^{(s)}_k\cr
&\copy\rl& &\copy\ru&       \cr
&     &q^{(s+1)}_k
   &\omit$=q^{(s)}_k+(e^{(s)}_k-
     e^{(s+1)}_{k+1})$\hfil\hidewidth
                 \cr}%end halign
}% end vbox element
%CGL insertion of qquads will shift
%CGL the box to the left
\qquad\qquad  \qquad
}% end displaylines
$$

\item {Continued fractions}:
```

```
\gdef\cf{\mathop{\grkop \Phi}}
{\newskip\centering
\centering=0pt plus1000pt minus 1000pt
$$\eqalignno{
1+\cf_{k=1}^n{a_k\over b_k}
&{}\buildrel{\rm def}\over=
 1+{a_1\over\displaystyle b_1+
   {\strut a_2\over\strut
    \vrule height3ex width0pt\relax
    \displaystyle b_2 +
     \lower2.0ex\hbox{$\ddots\,
      \lower1.25ex\hbox{$+
      {\displaystyle a_{n-1}\over
        \displaystyle b_{n-1}+
                 {\strut a_n\over
           \displaystyle b_n}}$}
                        $}
    }
   }\cr
\omit {\rm with (space saving)
     variant notations:}\hidewidth\cr
&{}\buildrel{\rm\phantom{def}}\over=
1+{a_1\,
\smash{\vrule depth1ex}\vrule height2ex
\over\strut\vrule\,b_1}+{a_2\,
\smash{\vrule depth1ex}\vrule height2ex
\over\strut\vrule\,b_2}+\cdots+{a_n\,
\smash{\vrule depth1ex}\vrule height2ex
\over\strut\vrule\,b_n}\cr
%
&{}\buildrel{\rm\phantom{def}}\over=
1+
{a_1\over\textstyle\strut
    \vrule height2.5ex width0pt
    b_1\,+\,}
{a_2\over\textstyle\strut
    \vrule height2.5ex width0pt
    b_2\,+\,}
    \cdots
{a_n\over\textstyle\strut
    \vrule height2.5ex width0pt
    b_n}
\cr}%end\eqalignno
$$}%
%
\item Reduction to Hessenberg form via
lower triangular similarity transformation
(Wilkinson, 1965:357):
$$\displaylines{\indent
\bordermatrix{&      &\rm A &      \cr
              &\times&\times&\times\cr
              &\times&\times&\times\cr
              &\times&\times&\times\cr}
\bordermatrix{& &\rm N & \cr
              &1&      & \cr
              &0&1     & \cr
              &0&\times&1\cr}\hfill\cr
\hfill=
\bordermatrix{& &\rm N & \cr
              &1&      & \cr
              &0&1     & \cr
              &0&\times&1\cr}
\bordermatrix{&      &\rm H &      \cr
              &\times&\times&\times\cr
              &\times&\times&\times\cr
              &0     &\times&\times\cr}
}$$
%
\item {Partitioning} (Wilkinson, 1965:291):
$$P_r=\left(\vcenter{
    \offinterlineskip\tabskip0pt
    \halign{
      \vrule height3ex depth1ex width 0pt
      \hfil$\enspace#\enspace$\hfil
      \vrule width.1pt\relax
      &\hfil$\enspace#\enspace$\hfil\cr
      I_{n-r}&0\cr
      \noalign{\hrule height.1pt\relax}
      0      &I-2v_rv_r^T\cr}
          }\right)          $$
%
\item Braces and matrices (Wilkinson, 1965:199):
$$\vcenter{
  \hbox{${\scriptstyle\phantom{n{-}}p}
    \left\{\vrule height4.5ex width0pt
         depth 0pt\right.$}\vglue3ex\relax
  \hbox{${\scriptstyle n{-}p}
    \left\{\vrule height2.5ex width0pt
         depth 0pt\right.\kern2pt$}
         \vglue1ex\relax
      }
\bordermatrix{&\multispan4{\enspace\hfil
$\overbrace{\vrule height0pt width10ex
   depth0pt}^p$}\hfil
             &\multispan3{\enspace\hfil
$\overbrace{\vrule height0pt width7.5ex
   depth0pt}^{n-p}$}\hfil\cr
\noalign{\kern-.25\baselineskip}
&\times&\times&\times&\times&\times&
 \times&\times\cr
&0     &\times&\times&\times&\times&
 \times&\times\cr
&0     &0     &\times&\times&\times&
 \times&\times\cr
&0     &0     &0     &\times&\times&
 \times&\times\cr
&0     &0     &0     &0     &\times&
 \times&\times\cr
&0     &0     &0     &0     &\times&
 \times&\times\cr
&0     &0     &0     &0     &\times&
 \times&\times\cr
}$$

%
\item Matrices, braces, (dotted)
 partitioning and icons
 (space efficient variant):
```

```
%The simplest way is to make the 22-element
%separate, and measure the sizes.
%Subsequently one easily couples these
%sizes to the sizes of the braces.
%Hard things: automatic coupling,
%          vertical dotted lines,
%          inner use \noalign.

\def\vdts{\vbox{\baselineskip4pt
  \lineskiplimit0pt
  \vglue2pt\hbox{.}\hbox{.}\hbox{.}}}%
$$
\vcenter{\offinterlineskip%No interline
%                space in between parts
\halign{\hfil$#$&\hfil$#$\hfil\cr%2-column
%first row with braces, element 11 empty
{}&\hfil\enspace\mathop{\hbox to.9cm%
   {\downbracefill}}\limits^{\vbox{\hbox{
               $\scriptstyle p$}\kern2pt}}
        \enspace\hfil\mathop{\hbox to.6cm%
   {\downbracefill}}\limits^{\vbox{\hbox to
  0pt{\hss$\scriptstyle n-p$\hss}\kern2pt}}%
        \enspace\hfil\cr  % end first row
%Separation between first (border) row and
%second row
\noalign{\vglue1ex}
%first column with braces, 21 element
\vcenter{\vfil
   \hbox{${\scriptstyle p}\left\{\vbox
   to.8cm{}\right.$}\vfil\vglue2ex\vfil
   \hbox{\llap{$\scriptstyle n{}$}%
   ${\scriptstyle p}\left\{\vbox to.5cm{}
   \right.$}\vfil}
&%22-element is the matrix proper
\left(\vcenter{\offinterlineskip
\halign{\hfil$#$\hfil&\hfil$#$\hfil&
\hfil$#$\hfil&\hfil$#$\hfil
\tabskip=.5\tabskip&\vdts#&
\tabskip=2\tabskip
\hfil$#$\hfil&\hfil$#$\hfil&
\hfil$#$\hfil\cr%end template
\times&\times&\times&\times&&\times&
 \times&\times\cr
0     &\times&\times&\times&&\times&
 \times&\times\cr
0     &0     &\times&\times&&\times&
 \times&\times\cr
0     &0     &0     &\times&&\times&
 \times&\times\cr
\noalign{\vglue1ex}
\multispan8\dotfill\cr
0     &0     &0     &0     &&\times&
 \times&\times\cr
0     &0     &0     &0     &&\times&
 \times&\times\cr
0     &0     &0     &0     &&\times&
\times&\times\cr}%end halign (22)
}%end vcenter
\right)\cr %end 22-element
```

```
%Separation between last (border) row
%and previous (second row)
\noalign{\vglue1ex}
{}&\hfil\enspace\mathop{\hbox to.9cm{%
   \upbracefill}}\limits_{\vbox{\kern2pt
                     \icurt{4ex}{2ex}}}
   \enspace\hfil
   \mathop{\hbox to.6cm{%
   \upbracefill}}\limits_{\vbox{\kern2pt
     \icmat{4ex}{1.5ex}}}\enspace\hfil%
\cr  % end last row
}%end halign
}%end vbox
$$
%
%References
%Addison-Wesley. Micro-TeX.
%Wilkinson, J.H. (1963):
%   The Algebraic Eigenvalue problem.
%Swanson, E. (1986):
%   Mathematics into Type. AMS.
%Doob, M. (1989): Gentle TeX.
%Hendrickson, Amy (priv.comm)
%Schwarz, Rutishauser, Stiefel (1972):
%   Matrizen numeriek
%
\ei

}
```