* * * * * * * * * * *

BUBBLES: A TEXTENSION IN SEARCH OF A TEXPE

Timothy Murphy
Trinity College Dublin

Preamble

The bubble notation used in particle physi identical in format with a notation for tensors to the relativist Roger Penrose. This "langu is strictly formalised, with the possible diagr governed by a few simple rules. In effect, a diag consists of a number of "boxes", joined togethe lines. A TEX extension is proposed, in which Pen or bubble diagrams can be displayed in exactly same way as mathematical formulae.

Analysis of the notation

There are 2 elements in the diagrams.

First, there are the **tensors** or **bubbles**. Eac these consists of a "container" of some kind, usu a rectangle or a circle. Inside there may be an id tifying name or symbol.

Emerging from the perimeters of these contai are the second element of the diagrams, the a An arm may join 2 tensors; or it may extend to edge of the diagram.

The relative positions of the arms on a tensor significant, e.g. if the tensor T is represented rectangle, with 2 upper arms and 1 lower, then left upper arm must be distinguished from the r upper arm; and both are quite different from lower arm.

(If only that sought-for TEXpert could tell me to replace this pedantic description of a box magical control sequence ... !)

Meaning of the Penrose notation

Although not strictly necessary, it may hel I explain, very briefly, the interpretation of Penrose notation for tensors.

In the classical (Einstein) notation, a tensor type $(1,2)$ (for example) is denoted by

$$T^i_{jk}$$

Here i, j and k are "dummy suffixes", so that e

$$T^a_{bc}$$

represents exactly the same tensor.

In the Penrose notation, T is incarcerated box, with 1 upper arm (corresponding to the u index i) rising from the top of the box, and 2 l arms (corresponding to the lower indices j an descending from the bottom of the box.

The joining of arms on 2 tensors (or on the s tensor) in a Penrose diagram corresponds to the **traction** of the corresponding indices—denote the Einstein notation simply by repetition of the index. For example, the tensor

$$S^i_j T^k_{il},$$

obtained by contracting the upper index of S with an upper index of T, is represented by a diagram with 2 boxes, one for S with a single upper and a single lower arm, and one for T as above. The upper arm of S is joined to the left-hand lower arm of T, to represent the contraction. The remaining arms extend upwards or downwards (as the case may be) to the edge of the diagram. Thus the resulting tensor is of type $(1,2)$, with 1 upper arm, corresponding to k, and 2 lower arms, corresponding to j and l.

The relative positions of S and T are immaterial. They may be side-by-side; or S may be above T, or vice versa. The choice is made by the author on aesthetic or other grounds, e.g. to minimise the entanglement of arms. In this case, for example, the natural solution would be to place S above T.

Proposed macro definition

We shall use the control sequence `\T` for rectangular tensors with upper and lower arms.

The control sequence `\T` has 3 parameters: First, the name or symbol (possibly null) of the tensor. Next the upper arms, followed by a comma, and then the lower arms, e.g.

```
\T S i,j    \T T i,jk .
```

In other words, the definition starts

```
\.def\T#1#2,#3 {...}.
```

Turning to the 2nd and 3rd parameters, each of these consists of an 'hlist'. But the members of each list are in a strange new font, having the property that all letters in this font are represented by dots, of width (say) 1 em. The dots are positioned on the top and bottom edges of the box, and serve

(1) to determine the width of the box,
(2) to define the ends of the arms, and
(3) to label these arms.

Finally—and this is the difficult part—wherever a letter occurs twice in the parameter hlists of tensors, the corresponding points must be joined by lines. These joining arms must leave each tensor at right angles to the box—their paths being defined thereafter by cubic splines, a la METAFONT.

Other tensor shapes

We would like at least 2 other tensor formats.

First we should like to allow arms to emerge from all 4 edges of a rectangular box. This might be described by a control sequence

```
\q#1#2,#3,#4,#5
```

with the hlists corresponding to parameters 2 to 5 describing the arms emerging from the top, bottom, left and right of the box.

And we would also like to have circular tensors (or bubbles), with the property that arms could emerge in any radial direction, provided the cyclic order around the tensor was maintained. In general the arms would try to reach their destination as directly as possible. The control sequence for such tensors might be

```
\C#1#2
```

where `#1` is the name of the tensor, as usual, while `#2` is a single hlist giving the arms in (say) anticlockwise order.

Implementation

The first part of the construction, drawing the boxes, can be accomplished simply enough within TEX. We have not specified the horizontal or vertical spacing between boxes, but that is a mere detail.

The main problem arises, of course, when it comes to drawing the arms that link the boxes. Clearly that cannot be accommodated within standard TEX. However, the basic idea—joining 2 given points by a curve leaving the end-points in specified directions—is the very stuff of METAFONT.

The arms might need a gentle nudge, certainly, to circumnavigate the boxes, e.g. to determine on which side of a box to go when joining an upper arm to a lower on the same box.

But these are minor details. The essential question is: how can we save the hlists of "indices" appearing in the tensor parameter lists, and then join repeated indices by curved lines?

In conclusion

There have been several suggestions—some implemented—for interspersing TEX output with computerised graphics. It should be emphasised that our proposal is rather different from these.

We are asking for both more and less. More, in that we would like our bubbles to be device-independent in the same sense as TEX itself. Less, in that our graphical requirement is modesty itself—merely the ability to draw the crudest of boxes, and to join them with the simplest of curves.

Yet it may be that this kind of extension—the incorporation of graphical elements into the actual text—will prove more significant in the long run than the ability to draw the most beautiful of diagrams.

* * * * * * * * * * *