

APPENDIX A

Output Samples from a Paper by David Eck

The pages given here are various renditions of one page from a paper by David Eck of Dartmouth College, a first-time user of $\text{T}_{\text{E}}\text{X}$ and $\text{A}_{\text{M}}\text{S}-\text{T}_{\text{E}}\text{X}$. (He describes his experience on page 127.) One page was selected from his paper, and run through $\text{T}_{\text{E}}\text{X}$ at three locations: the American Mathematical Society, Massachusetts Institute of Technology, and Stanford. The composed file was then output to whatever devices were available at each location. The samples are given in the following order.

- input file listing;
- Florida Data Model BNY dot matrix printer, 128 dots/inch;
- Xerox XGP, 200 dots/inch;
- Benson-Varian 9211, 200 dots/inch;
- Canon Laser Beam Printer, 240 dots/inch;
- Xerox Dover, 384 dots/inch;
- Alphatype CRS, 5300 dots/inch.

At the AMS a few rudimentary timing estimates were made. For such a small sample, these times probably represent an upper bound, rather than a true measure of time required to process this type of material. The computer involved is a DEC 2060 with 512K words of memory. Output to the Varian is controlled by a spooler, which removes the processing from user control once a job has been released to the queue. No times were obtained for output to any device but the Varian, but an attempt will be made to obtain and publish such information in a later issue of TUGboat.

- $\text{T}_{\text{E}}\text{X}$: CPU time of < 6 sec. included loading the program and reading all font and macro header files; net time required for processing the one page of input was probably < 3 sec.
- release to Varian spooler: 1 CPU sec.
- printing on Varian: < 4 CPU sec.

Output from another device, a Compugraphic 8600, is shown and described in the article by Ralph Stromquist, page 51. The macro packages and other items by Lynne Price and Pat Milligan (pages 87-126) were output on a Versatec, which is essentially similar to the Varian 9211. The two articles by Lawson, Zabala and Díaz (pages 20-47) were output on a Dover, as were the macro package descriptions by Max Díaz and Arthur Keller. The body of this newsletter was output on the Varian at the AMS.

*This is eck secl

\chapterbegin{\section 1: The Functors $(\cdot)^n_k$ }

\definition Definition 1.1\\ If $n \geq 0$ and $k \geq 0$ are integers, we define a functor $(\cdot)^n_k$ as follows:
 If M is a C^∞ manifold, then $(\cdot)^n_k M = \left\{ \varphi : \mathbb{R}^n \rightarrow M \mid \varphi(0) = p \right\}$ where $\varphi(0)$ is the k -jet of φ at 0 . If $f: M \rightarrow N$ is a smooth map, then $(\cdot)^n_k f: (\cdot)^n_k M \rightarrow (\cdot)^n_k N$ is defined by $(\cdot)^n_k f(\varphi) = \varphi \circ f$.

Note that $(\cdot)^n_k M$ is a fiber bundle over M , with projection $\pi: (\cdot)^n_k M \rightarrow M$ given by $\pi(\varphi) = \varphi(0)$. In particular, $(\cdot)^1_1 M$ is just the tangent bundle $T M$. This is clear if we consider a tangent vector at a point p of M to be an equivalence class of curves in M which agree up to the first order at p . We will need to know the following basic properties of the functor $(\cdot)^n_k$. They are easy to establish, and we omit the proofs.

\theorem Theorem 1.2\\ a) If M and N are manifolds, then $(\cdot)^n_k M \times (\cdot)^n_k N$ and $(\cdot)^n_k (M \times N)$ are naturally equivalent.
 b) If M is a manifold, then $(\cdot)^m_k M$ and $(\cdot)^{m+k}_k M$ are naturally equivalent. \QED

We note that by naturality here, we mean, in a), that given any maps $f: M \rightarrow N$, $g: N \rightarrow N'$, the diagram $(M \times N)^n_k \rightarrow (M \times N')^n_k \rightarrow (M \times N)^n_k$ commutes, and similarly for b).

\par It may be useful to see what the map $\Psi: (\cdot)^m_k M \rightarrow (\cdot)^{m+k}_k M$ looks like in coordinates:

If y_1, \dots, y_s are local coordinates on M , we get local coordinates y^α , $\alpha = (1, \dots, n)$ with $|\alpha| \leq k$, on $(\cdot)^n_k M$ by $y^\alpha(\varphi) = \frac{\partial^\alpha x}{\partial t^k}(\varphi(0))$. By extension, we have coordinates $y^{\alpha; \beta}$ on $(\cdot)^m_k M$ and $\overline{y^{\beta; \alpha}}$ on $(\cdot)^{m+k}_k M$, where $\beta = (1, \dots, m)$ with $|\beta| \leq k$. The map Ψ is given in these coordinates by $\Psi(\varphi) = \overline{y^{\beta; \alpha}}(\varphi) = y^{\alpha; \beta}(\varphi)$.

We will use the following notation: \mathcal{M}^n will denote the set of all smooth maps $\mathbb{R}^n \rightarrow M$. Whenever we consider \mathcal{M}^n as a topological space, we will always use the C^∞ topology. If $p \in M$, then we denote the constant map $\mathbb{R}^n \rightarrow M$ which sends each element of \mathbb{R}^n to p by A_p .

Florida Data Model BNY

§1: THE FUNCTORS $(\cdot)_k^n$

DEFINITION 1.1: If $n \geq 0$ and $k \geq 0$ are integers, we define a functor $(\cdot)_k^n$ as follows: If M is a C^∞ manifold, then

$$M_k^n = \{ j_k(\varphi)_0 \mid \varphi: \mathbb{R}^n \rightarrow M \}$$

where $j_k(\varphi)_0$ is the k -jet of φ at 0. If $f: M \rightarrow N$ is a smooth map, then $f_k^n: M_k^n \rightarrow N_k^n$ is defined by

$$f_k^n(j_k(\varphi)_0) = j_k(f \circ \varphi)_0.$$

Note that M_k^n is a fiber bundle over M , with projection $\pi: M_k^n \rightarrow M$ given by $\pi(j_k(\varphi)_0) = \varphi(0)$. In particular, M_1^1 is just the tangent bundle TM . This is clear if we consider a tangent vector at a point p of M to be an equivalence class of curves in M which agree up to the first order at p . We will need to know the following basic properties of the functor $(\cdot)_k^n$. They are easy to establish, and we omit the proofs.

THEOREM 1.2: a) If M and N are manifolds, then $M_k^n \times N_k^n$ and $(M \times N)_k^n$ are naturally equivalent.

b) If M is a manifold, then $(M_k^n)_l^m$ and $(M_l^m)_k^n$ are naturally equivalent. ■

We note that by naturality here, we mean, in a), that given any maps $f: M \rightarrow M'$ and $g: N \rightarrow N'$, the diagram

$$\begin{array}{ccc} (M \times N)_k^n & \xrightarrow{(f \times g)_k^n} & (M' \times N')_k^n \\ \downarrow & & \downarrow \\ (M_k^n \times N_k^n) & \xrightarrow{f_k^n \times g_k^n} & (M'_k^n \times N'_k^n) \end{array}$$

commutes, and similarly for b).

It may be useful to see what the map $\psi: (M_k^n)_l^m \rightarrow (M_l^m)_k^n$ looks like in coordinates:

If y_1, \dots, y_n are local coordinates on M , we get local coordinates y_i^α , $\alpha = (\alpha_1, \dots, \alpha_n)$ with $|\alpha| \leq k$, on M_k^n by

$$y_i^\alpha(j_k(\varphi)_0) = \frac{\partial^\alpha y_i \circ \varphi}{\partial x^\alpha}(0).$$

§1: THE FUNCTORS $(\cdot)_k^n$

DEFINITION 1.1: If $n \geq 0$ and $k \geq 0$ are integers, we define a functor $(\cdot)_k^n$ as follows: If M is a C^∞ manifold, then

$$M_k^n = \{ j_k(\varphi)_0 \mid \varphi : \mathbb{R}^n \rightarrow M \}$$

where $j_k(\varphi)_0$ is the k -jet of φ at 0. If $f : M \rightarrow N$ is a smooth map, then $f_k^n : M_k^n \rightarrow N_k^n$ is defined by

$$f_k^n(j_k(\varphi)_0) = j_k(f \circ \varphi)_0.$$

Note that M_k^n is a fiber bundle over M , with projection $\pi : M_k^n \rightarrow M$ given by $\pi(j_k(\varphi)_0) = \varphi(0)$. In particular, M_1^1 is just the tangent bundle TM . This is clear if we consider a tangent vector at a point p of M to be an equivalence class of curves in M that agree up to the first order at p . We will need to know the following basic properties of the functor $(\cdot)_k^n$. They are easy to establish, and we omit the proofs.

THEOREM 1.2: a) If M and N are manifolds, then $M_k^n \times N_k^n$ and $(M \times N)_k^n$ are naturally equivalent.

b) If M is a manifold, then $(M_k^n)_\ell^m$ and $(M_\ell^m)_k^n$ are naturally equivalent. ■

We note that by naturality here, we mean, in a), that given any maps $f : M \rightarrow M'$ and $g : N \rightarrow N'$, the diagram

$$\begin{array}{ccc} (M \times N)_k^n & \xrightarrow{(f \times g)_k^n} & (M' \times N')_k^n \\ \downarrow & & \downarrow \\ (M_k^n \times N_k^n) & \xrightarrow{f_k^n \times g_k^n} & (M'_k^n \times N'_k^n) \end{array}$$

commutes, and similarly for b).

It may be useful to see what the map $\Psi : (M_k^n)_\ell^m \rightarrow (M_\ell^m)_k^n$ looks like in coordinates:

If y_1, \dots, y_n are local coordinates on M , we get local coordinates y_i^α , $\alpha = (\alpha_1, \dots, \alpha_n)$ with $|\alpha| \leq k$, on M_k^n by

$$y_i^\alpha(j_k(\varphi)_0) = \frac{\partial^\alpha y_i \circ \varphi}{\partial x^\alpha}(0).$$

By extension, we have coordinates $y_i^{\alpha;\beta}$ on $(M_k^n)_\ell^m$ and $\bar{y}_i^{\beta;\alpha}$ on $(M_\ell^m)_k^n$, where $\beta = (\beta_1, \dots, \beta_m)$ with $|\beta| \leq \ell$. The map Ψ is given in these coordinates by $\Psi((y^{\alpha;\beta})) = (\bar{y}^{\beta;\alpha})$ where $\bar{y}_i^{\beta;\alpha} = y_i^{\alpha;\beta}$.

§1: THE FUNCTORS $(\cdot)_k^n$

DEFINITION 1.1: If $n \geq 0$ and $k \geq 0$ are integers, we define a functor $(\cdot)_k^n$ as follows: If M is a C^∞ manifold, then

$$M_k^n = \{ j_k(\varphi)_0 \mid \varphi: \mathbb{R}^n \rightarrow M \}$$

where $j_k(\varphi)_0$ is the k -jet of φ at 0. If $f: M \rightarrow N$ is a smooth map, then $f_k^n: M_k^n \rightarrow N_k^n$ is defined by

$$f_k^n(j_k(\varphi)_0) = j_k(f \circ \varphi)_0.$$

Note that M_k^n is a fiber bundle over M , with projection $\pi: M_k^n \rightarrow M$ given by $\pi(j_k(\varphi)_0) = \varphi(0)$. In particular, M_1^1 is just the tangent bundle TM . This is clear if we consider a tangent vector at a point p of M to be an equivalence class of curves in M which agree up to the first order at p . We will need to know the following basic properties of the functor $(\cdot)_k^n$. They are easy to establish, and we omit the proofs.

THEOREM 1.2: a) If M and N are manifolds, then $M_k^n \times N_k^n$ and $(M \times N)_k^n$ are naturally equivalent.

b) If M is a manifold, then $(M_k^n)_\ell^m$ and $(M_\ell^m)_k^n$ are naturally equivalent. ■

We note that by naturality here, we mean, in a), that given any maps $f: M \rightarrow M'$ and $g: N \rightarrow N'$, the diagram

$$\begin{array}{ccc} (M \times N)_k^n & \xrightarrow{(f \times g)_k^n} & (M' \times N')_k^n \\ \downarrow & & \downarrow \\ (M_k^n \times N_k^n) & \xrightarrow{f_k^n \times g_k^n} & (M')_k^n \times (N')_k^n \end{array}$$

commutes, and similarly for b).

It may be useful to see what the map $\Psi: (M_k^n)_\ell^m \rightarrow (M_\ell^m)_k^n$ looks like in coordinates:

If y_1, \dots, y_s are local coordinates on M , we get local coordinates y_i^α , $\alpha = (\alpha_1, \dots, \alpha_n)$ with $|\alpha| \leq k$, on M_k^n by

$$y_i^\alpha(j_k(\varphi)_0) = \frac{\partial^\alpha y_i \circ \varphi}{\partial x^\alpha}(0).$$

§1: THE FUNCTORS $(\cdot)_k^n$

DEFINITION 1.1: If $n \geq 0$ and $k \geq 0$ are integers, we define a functor $(\cdot)_k^n$ as follows: If M is a C^∞ manifold, then

$$M_k^n = \{ j_k(\varphi)_0 \mid \varphi: \mathbb{R}^n \rightarrow M \}$$

where $j_k(\varphi)_0$ is the k -jet of φ at 0. If $f: M \rightarrow N$ is a smooth map, then $f_k^n: M_k^n \rightarrow N_k^n$ is defined by

$$f_k^n(j_k(\varphi)_0) = j_k(f \circ \varphi)_0.$$

Note that M_k^n is a fiber bundle over M , with projection $\pi: M_k^n \rightarrow M$ given by $\pi(j_k(\varphi)_0) = \varphi(0)$. In particular, M_1^1 is just the tangent bundle TM . This is clear if we consider a tangent vector at a point p of M to be an equivalence class of curves in M that agree up to the first order at p . We will need to know the following basic properties of the functor $(\cdot)_k^n$. They are easy to establish, and we omit the proofs.

THEOREM 1.2: a) If M and N are manifolds, then $M_k^n \times N_k^n$ and $(M \times N)_k^n$ are naturally equivalent.

b) If M is a manifold, then $(M_k^n)_\ell^m$ and $(M_\ell^m)_k^n$ are naturally equivalent. ■

We note that by naturality here, we mean, in a), that given any maps $f: M \rightarrow M'$ and $g: N \rightarrow N'$, the diagram

$$\begin{array}{ccc} (M \times N)_k^n & \xrightarrow{(f \times g)_k^n} & (M' \times N')_k^n \\ \downarrow & & \downarrow \\ (M_k^n \times N_k^n) & \xrightarrow{f_k^n \times g_k^n} & (M')_k^n \times (N')_k^n \end{array}$$

commutes, and similarly for b).

It may be useful to see what the map $\Psi: (M_k^n)_\ell^m \rightarrow (M_\ell^m)_k^n$ looks like in coordinates:

If y_1, \dots, y_s are local coordinates on M , we get local coordinates y_i^α , $\alpha = (\alpha_1, \dots, \alpha_n)$ with $|\alpha| \leq k$, on M_k^n by

$$y_i^\alpha(j_k(\varphi)_0) = \frac{\partial^\alpha y_i \circ \varphi}{\partial x^\alpha}(0).$$

§1: THE FUNCTORS $(\cdot)_k^n$

DEFINITION 1.1: If $n \geq 0$ and $k \geq 0$ are integers, we define a functor $(\cdot)_k^n$ as follows: If M is a C^∞ manifold, then

$$M_k^n = \{ j_k(\varphi)_0 \mid \varphi : \mathbb{R}^n \rightarrow M \}$$

where $j_k(\varphi)_0$ is the k -jet of φ at 0. If $f : M \rightarrow N$ is a smooth map, then $f_k^n : M_k^n \rightarrow N_k^n$ is defined by

$$f_k^n(j_k(\varphi)_0) = j_k(f \circ \varphi)_0.$$

Note that M_k^n is a fiber bundle over M , with projection $\pi : M_k^n \rightarrow M$ given by $\pi(j_k(\varphi)_0) = \varphi(0)$. In particular, M_1^1 is just the tangent bundle TM . This is clear if we consider a tangent vector at a point p of M to be an equivalence class of curves in M that agree up to the first order at p . We will need to know the following basic properties of the functor $(\cdot)_k^n$. They are easy to establish, and we omit the proofs.

THEOREM 1.2: a) If M and N are manifolds, then $M_k^n \times N_k^n$ and $(M \times N)_k^n$ are naturally equivalent.

b) If M is a manifold, then $(M_k^n)_k^n$ and $(M_k^n)_k^n$ are naturally equivalent. ■

We note that by naturality here, we mean, in a), that given any maps $f : M \rightarrow M'$ and $g : N \rightarrow N'$, the diagram

$$\begin{array}{ccc} (M \times N)_k^n & \xrightarrow{(f \times g)_k^n} & (M' \times N')_k^n \\ \downarrow & & \downarrow \\ (M_k^n \times N_k^n) & \xrightarrow{f_k^n \times g_k^n} & (M')_k^n \times (N')_k^n \end{array}$$

commutes, and similarly for b).

It may be useful to see what the map $\Psi : (M_k^n)_k^n \rightarrow (M_k^n)_k^n$ looks like in coordinates:

If y_1, \dots, y_s are local coordinates on M , we get local coordinates y_i^α , $\alpha = (\alpha_1, \dots, \alpha_n)$ with $|\alpha| \leq k$, on M_k^n by

$$y_i^\alpha(j_k(\varphi)_0) = \frac{\partial^\alpha y_i \circ \varphi}{\partial x^\alpha}(0).$$

Alphatype CRS

§1: THE FUNCTORS $(\cdot)_k^n$

DEFINITION 1.1: If $n \geq 0$ and $k \geq 0$ are integers, we define a functor $(\cdot)_k^n$ as follows: If M is a C^∞ manifold, then

$$M_k^n = \{j_k(\varphi)_0 \mid \varphi: \mathbb{R}^n \rightarrow M\}$$

where $j_k(\varphi)_0$ is the k -jet of φ at 0. If $f: M \rightarrow N$ is a smooth map, then $f_k^n: M_k^n \rightarrow N_k^n$ is defined by

$$f_k^n(j_k(\varphi)_0) = j_k(f \circ \varphi)_0.$$

Note that M_k^n is a fiber bundle over M , with projection $\pi: M_k^n \rightarrow M$ given by $\pi(j_k(\varphi)_0) = \varphi(0)$. In particular, M_1^1 is just the tangent bundle TM . This is clear if we consider a tangent vector at a point p of M to be an equivalence class of curves in M which agree up to the first order at p . We will need to know the following basic properties of the functor $(\cdot)_k^n$. They are easy to establish, and we omit the proofs.

THEOREM 1.2: a) If M and N are manifolds, then $M_k^n \times N_k^n$ and $(M \times N)_k^n$ are naturally equivalent.

b) If M is a manifold, then $(M_k^n)_l^m$ and $(M_l^m)_k^n$ are naturally equivalent. ■

We note that by naturality here, we mean, in a), that given any maps $f: M \rightarrow M'$ and $g: N \rightarrow N'$, the diagram

$$\begin{array}{ccc} (M \times N)_k^n & \xrightarrow{(f \times g)_k^n} & (M' \times N')_k^n \\ \downarrow & & \downarrow \\ (M_k^n \times N_k^n) & \xrightarrow{f_k^n \times g_k^n} & (M')_k^n \times (N')_k^n \end{array}$$

commutes, and similarly for b).

It may be useful to see what the map $\Psi: (M_k^n)_l^m \rightarrow (M_l^m)_k^n$ looks like in coordinates:

If y_1, \dots, y_s are local coordinates on M , we get local coordinates y_i^α , $\alpha = (\alpha_1, \dots, \alpha_n)$ with $|\alpha| \leq k$, on M_k^n by

$$y_i^\alpha(j_k(\varphi)_0) = \frac{\partial^\alpha y_i \circ \varphi}{\partial x^\alpha}(0).$$